THE VARIATIONAL MESHLESS METHOD: THEORY AND APPLICATIONS

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Abstract
The aim of this manuscript is to give a brief overview about the state of the art art of the Variational Meshless Method applied to the electromagnetic problems.

Index Terms – Meshless method, radial basis functions, variational method, waveguide and cavity modes.

I. INTRODUCTION

The Meshless Method is a well-known technique in various kind of physical problems (e.g. mechanics and fluidodynamics) [1].
Its introduction in the electromagnetic community is quite recent and is an event of the last decade [2]-[4].
During the last three years, the variational technique in conjunction with the meshless method has been proposed [5]-[8] and applied to various kind of problems permitting:

a) the computation of the propagating modes inside a waveguide of general shape by the solution of the scalar Helmholtz scalar problem
b) the evaluation of the dispersion diagram inside an inhomogeneous filled waveguide by the solution of the vector Helmholtz equation
c) the evaluation of the resonance modes inside an inhomogeneous filled cavity by solving the 3D Vector Helmholtz equation

In all this cases, the variational meshless method (VMM) seems to be a promising tool for the computation of a high number of modes with respect to the number of the unknowns (i.e. the problem size).

Respect to a) it is interesting to highlight that an automatic refinement technique has been developed in [6] to treat the regions of the domain in with there are the most rapid variations of the field (e.g. sharp corners).
The mode matching technique has also been applied to analyze the cascade of various waveguides with different shapes.
Respect to the point b) instead the use of the symmetries has also been exploited to reduce the problem size and thus improve the computation time of the simulation.

I. HOLLOW WAVEGUIDES SOLUTION

In [6] the problem of a hollow waveguide with a general cross section has been addressed. Fig. 1 shows a WR90 homogeneous waveguide simulated and the position of the collocation points where the unknowns are defined. With 248 collocation points and less than 0.5 s of simulation, the program gave 248 TE and 196 TM eigenvalues with the precision shown in Fig. 2

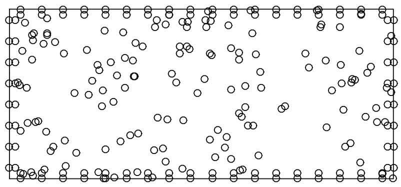

FIG. 1 – WR90 rectangular waveguide (22.86_10.16 mm2) and the collocation points used for the analysis by the variational meshless method.

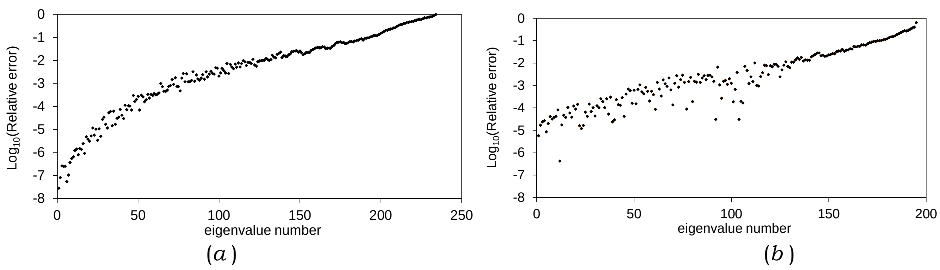

FIG. 2 – Error in the calculation of the cut-off frequency of a WR90 rectangular waveguide by the variational meshless method with 248 collocation points: (a) TE modes; (b) TM modes.

In Fig. 3 is shown a double ridge homogeneous waveguide simulated and the position of the collocation points before and after the automatic refinement. As can be seen the algorithm adds all the additional points around the sharp corners as expected.
In Fig. 4, the convergence of the algorithm is shown for the first 16 steps. All the results were in agreement with HFSS and the difference between the two simulators was less the 0.5% over the cut-off frequencies.

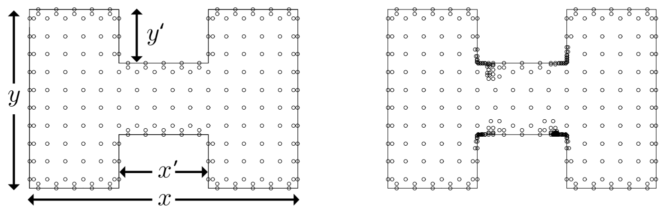

FIG. 3 –Double Ridge Waveguide (*x*=30mm, *y*=20mm, *x*'=10mm, and *y*'=6mm) and the collocation points used for the analysis by the variational meshless method before and after the refinement.

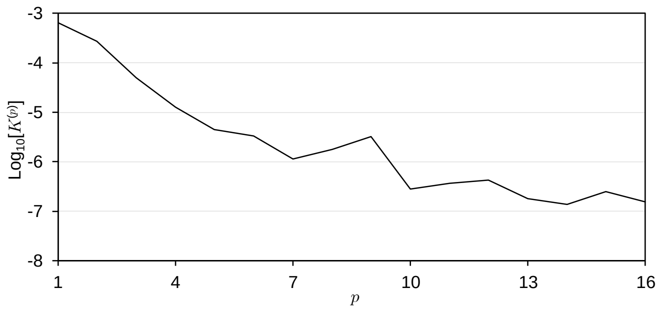

FIG. 4 – Convergence study of the method in the case of Fig.3 for the first 16 refinement steps.

II. DISPERSION DIAGRAM OF INHOMOGENEOUS WAVEGUIDES

In [7] the theory of [8] has been extended to compute the dispersion diagram of an inhomogeneous filled waveguide. In Fig. 5 an example is reported. The simulation needed 7.44 s to compute the initial solution and the reported 28 in frequency points.

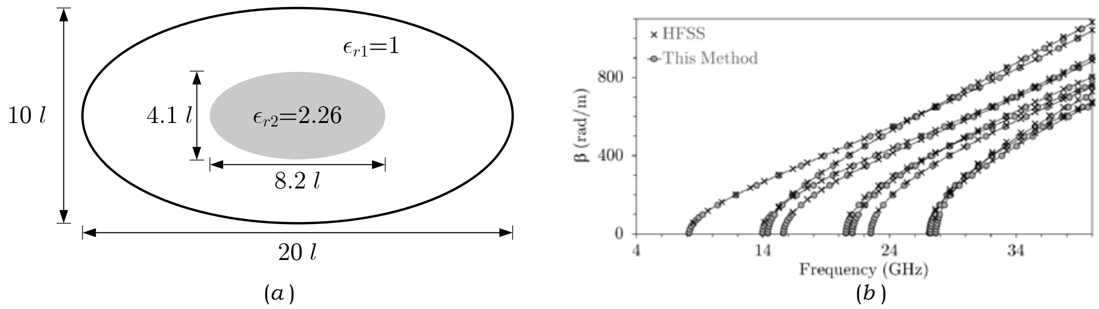

FIG. 5 – Elliptic inhomogeneous waveguide. (a) Geometry of the structure (l = 1 mm). (b) Dispersion diagram calculated by the variational meshless method (gray circles) compared with the HFSS simulation (black cross).

III. FINDING THE RESONANCE MODES OF A 3D RECTANGULAR CAVITY

In [8] the VMM has been applied to the evaluation of the resonance modes inside an inhomogeneous filled cavity. In Fig. 6 the case of an half-filled cavity is presented. As can be seen the difference between the two simulators was about the 0.1% over the resonance frequencies.

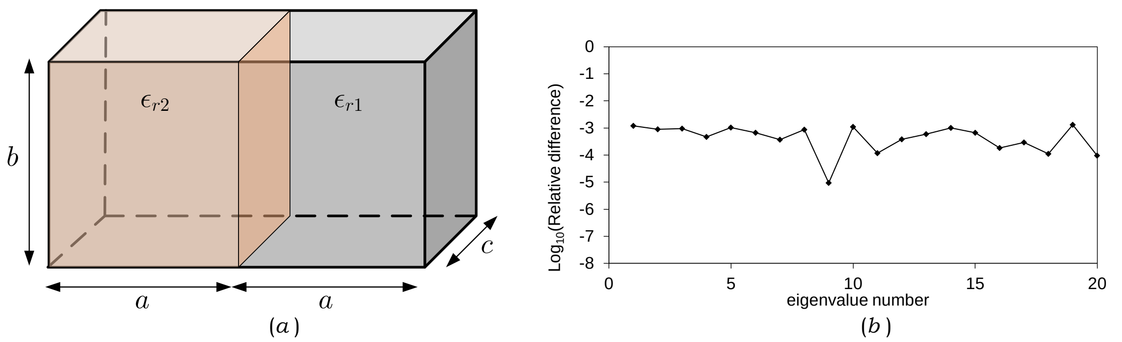

FIG. 6 –Half-filled rectangular cavity: (a) Geometry of the structure (a = 5 mm, b = 1 mm, c = 10 mm, ε_{r1} = 1, ε_{r2} = 2); (b) Relative difference between the present method and the results given by HFSS on the first 20 modes.

IV. CONCLUSION

The VMM seems to be a promising technique in the evaluation of the frequency response of guiding and resonant structures. In particular, it has been applied in various kind of 2D and 3D eigenproblems showing the ability to compute an high number of eigensolutions with a limited number of unknowns. Various examples were reported.

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