Array Antennas Diagnostics Through Phaseless Measurements and Sparsity-Promoting Approaches

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I. Introduction

The problem of characterizing a radiating source by using measurements setup as simple as possible and reduced measurement time gains great interest in the literature in different research areas. Amongst them, the problem of performing the diagnostics of an antenna [or near-field (NF) to far-field (FF) transformations] by using the least possible number of measurements is worth to be mentioned. Interestingly, the recently introduced compressive sensing (CS) theory [1] blends very well the above requirements. In fact, it allows the diagnosis of an array antenna by using a number of measurements (say M) much lower than the number of array elements (say N). Of course, because of the underlying CS theory, M is supposed to anyway be larger or even much larger than the number faulty elements (say S).

By following a different goal, phaseless measurements have also been the subject of intensive research. In fact, measuring a complex FF pattern requires a stable phase reference and accurate positioning of the probe in each measurement point, so that measurements may become problematic [2].

For all the above, the possible exploitation of CS for phaseless antenna diagnostics is of interest. Unfortunately, CS theory is established for linear problems, which is not the case at hand.

However, recent research activities at Università Mediterranea have shown the possibility of performing very accurate phaseless array antennas diagnosis by taking advantage from CS [3],[4]. In particular, three different strategies have been developed and assessed for the case of both linear and planar arrays, including cases where mutual coupling plays a relevant role.

II. Formulation of the Problem and the First Basic Strategy

A very common procedure to deal with array diagnostics is the definition of a virtual array as the difference between the 'gold' and the 'faulty' array under test (AUT) [5]. In this way, the resulting 'differential' array will be composed by few operative elements, the others being considered OFF, so that the exploitation of CS-based procedures is straightforward. In fact, the aim of the problem is to retrieve the number and location of the faulty elements that exactly correspond to the elements of the differential AUT.

Let denote with $E^G(u,v)$ and $E^F(u,v)$ the FF radiated by the 'gold' and 'faulty' AUT, respectively ($u = \beta d_x sin\theta cos\phi, v = \beta d_y sin\theta sin\phi$, with β the wavenumber, d_x and d_y the inter-element spacing, θ and ϕ respectively denoting the usual elevation and azimuth angles). Accordingly, $\Delta E(u,v) = E^G(u,v) - E^F(u,v)$ represents the FF radiated by the 'virtual differential' array.

Let now consider $\boldsymbol{e^G}$, $\boldsymbol{e^F}$ and $\boldsymbol{\Delta e}$ as the excitations vectors of the antennas constituting the 'gold', 'faulty' and 'differential' array, respectively.
When amplitude-only measurements are collected, the following relation holds:

$$|E^F(u_m,v_m)|^2 - |E^G(u_m,v_m)|^2 = |\Delta E(u_m,v_m)|^2 - 2Re\langle E^G(u_m,v_m), \Delta E(u_m,v_m)\rangle \quad (1)$$

that is non-linear in the unknown $\boldsymbol{\Delta e}$.
In order to apply CS-based procedures, one can note that:

i) as long as S is low, the energy of the signal ΔE is small with respect to the energy of E^G, so that the weight of the quadratic term on the right-hand side of (1) is negligible with respect to the linear one;

ii) the weight of $|\Delta E(u_m,v_m)|^2$ can be further reduced by increasing the contribution of the signal E^G by choosing (u_m,v_m) corresponding to high intensity signal.

By so doing, one is able to reduce the degree of non-linearity of the problem and the applicability of CS to the arising 'almost linear' problem is expected to be effective.
Therefore, the diagnostics problem from phaseless measurements can be formulated as a CP problem as:

$$\min\|\boldsymbol{\Delta e}\|_1 \qquad\qquad (2)$$
$$subject\ to\ \|\boldsymbol{\rho}\|_2 < \varepsilon$$

$\boldsymbol{\rho}$ being the residuals vector of eq.(1).
The proposed approach is general and can be applied to any kind of array, the only difference being the definition of the FF. As a matter of fact, if the array factor can be defined for the AUT, the expression #1 reported in Table 1 must be used in eq.(1), otherwise the FF can be expressed as the Fourier transform of the aperture field distribution of the AUT on a given aperture plane Σ, see expression #2 in Table 1.

CASE	FF EXPRESSION	UNKNOWN
#1: array factor is usable	$E(u,v) = \sum_{p=1}^{N_{antennas_x}} \sum_{q=1}^{N_{antennas_y}} I_{p,q} e^{j(pu+qv)}$	$\Delta I_{p,q}$: differential array elements excitations
#2: array factor cannot be used	$E(u,v) = \int_{\Sigma} E_{ap}(x,y)\, e^{j(ux+vy)} dxdy$	$\Delta E_{ap}(x,y)$: differential array aperture field

TABLE 1 – DEFINITIONS FOR THE FAR FIELD

It is worth to underly that, since the aperture field is a continuous distribution, the sparsity of the unknown $\Delta E_{ap}(x,y)$ may be compromised if it is still represented in the common 'pixel' representation basis. So, in order to take advantage of some sparsity-promotion-based procedures, two more effective solution procedures for such a case will be presented in the following Section.

III. TWO MORE EFFECTIVE SOLUTION APPROACHES FOR THE CASE OF NON-IDEAL ARRAYS

Adopting the representation of the FF in terms of aperture field distribution could allow the exploitation of a-priori information that can be profitably used for an effective applicability of CS procedures.

For example, in the case of an array of truncated waveguides, the aperture field for the differential problem is expected to be located essentially in correspondence with failures locations, thus occupying a region whose extension is at least equal to the physical waveguide's aperture. Hence, solving the problem as a total-variation (TV) norm-minimization one [6] seems to be a good option. In fact, the discrete gradient of ΔE_{ap} turns out to be 'sparse', i.e., different from zero in those pixels corresponding to the faulty waveguide's edges. Moreover, the addition of the derivatives along the principal and secondary diagonals $x = \pm y$ allows to counteract the information loss due to lack of phase information and, hence, achieve better regularization of the problem at hand. Accordingly, the optimization problem (2) recast as:

$$\min\left\{\left\|\boldsymbol{D_h \Delta E_{ap}}\right\|_1 + \left\|\boldsymbol{D_v \Delta E_{ap}}\right\|_1 + \left\|\boldsymbol{D_d}^{+}\boldsymbol{\Delta E_{ap}}\right\|_1 + \left\|\boldsymbol{D_d}^{-}\boldsymbol{\Delta E_{ap}}\right\|_1\right\} \quad \text{subject to } \|\boldsymbol{\rho}\|_2 < \varepsilon \tag{3}$$

wherein $\boldsymbol{D_h \Delta E_{ap}}$, $\boldsymbol{D_v \Delta E_{ap}}$, $\boldsymbol{D_d}^{+}\boldsymbol{\Delta E_{ap}}$, $\boldsymbol{D_d}^{-}\boldsymbol{\Delta E_{ap}}$ denote the vectors containing the forward differences of ΔE_{ap} along, respectively, the horizontal, vertical, principal-diagonal and secondary-diagonal directions.

The main drawback of the above strategy based on TV algorithm is that it may lead to high computational times in the case of electrically large arrays, that is when the degrees of freedom [7] are larger than the number of elements of the AUT. This limitation can be overcome by extending the adoption of the original 'active element pattern' [8] concept to the case of the aperture field, i.e., by defining an 'active element aperture pattern' (AAP_n) as the aperture field radiated by the AUT when its n-th antenna has unitary excitation while all the other as OFF, and then considering the following representation for ΔE_{ap}:

$$\Delta E_{ap}(x,y) = \sum_{n=1}^{N_{antennas}} \Delta\alpha_n AAP_n(x,y) \tag{4}$$

$\Delta\alpha_n$ being arbitrary coefficients that turn into the new unknowns of the problem, which recast as:

$$\min\|\boldsymbol{\Delta\alpha}\|_1 \quad \text{subject to } \|\boldsymbol{\rho}\|_2 < \varepsilon \tag{5}$$

Thanks to the new representation (4), the computational burden of the diagnostics approach goes back to being like that of the array-factor-based representation. Moreover, it allows to take into account mutual coupling among elements and mounting platform effects.

As a final comment, let note that, up to now, the diagnosis of ON-OFF defects has been successfully dealt with, but work is in progress about the detection of different kind of faults and the exploitation of near field data.

Numerical examples and comparisons amongst the different strategies will be shown at the Conference.

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