Shape Reconstruction via Equivalence Principles, Constrained Inverse Source Problems and Sparsity Promotion

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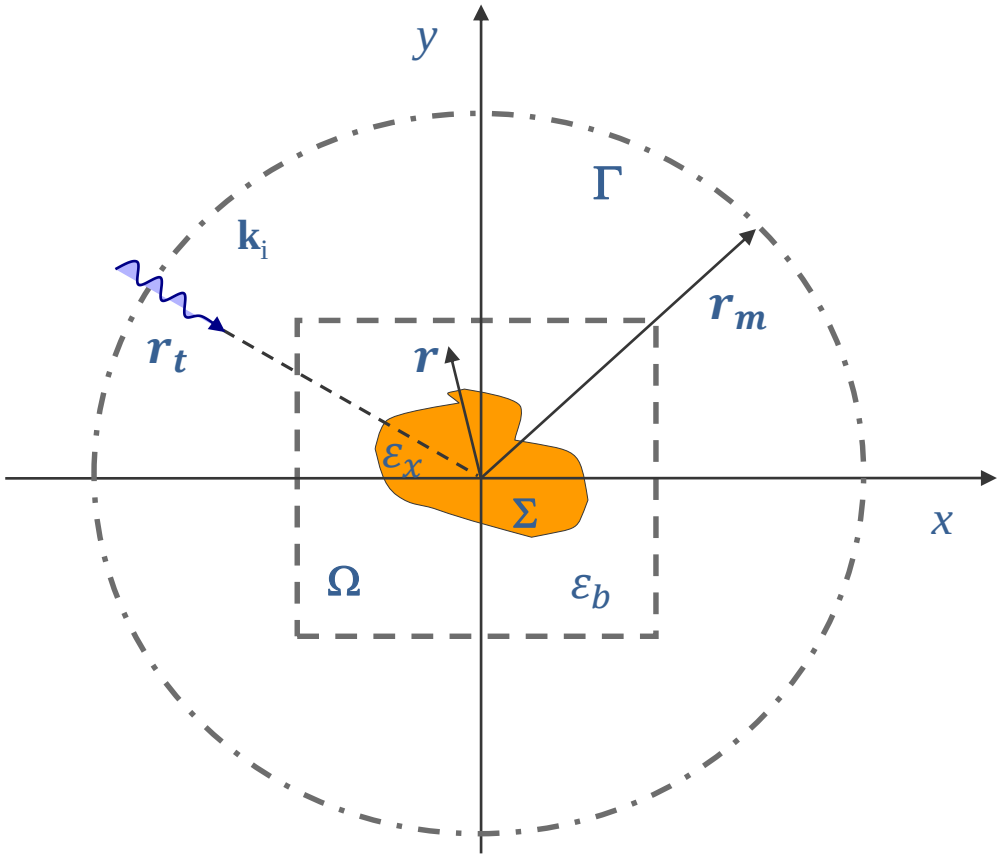
inverse scattering problem
(2D scalar case)



$$E_i(\boldsymbol{r}, \boldsymbol{r_t}), E_s(\boldsymbol{r_m}, \boldsymbol{r_t})$$

$$\boldsymbol{r} \in \Omega, \quad \boldsymbol{r_t}, \boldsymbol{r_m} \in \Gamma$$

$$\chi(\boldsymbol{r}) = \frac{\varepsilon_x(\boldsymbol{r})}{\varepsilon_b(\boldsymbol{r})} - 1$$

encodes target properties (e.m. parameters, shape)

inverse scattering problem

$$\boldsymbol{E_s}(\boldsymbol{r_m}, \boldsymbol{r_t}) = \int_{\Omega} G_b(\boldsymbol{r_m}, \boldsymbol{r'})\, \boldsymbol{W}(\boldsymbol{r'}, \boldsymbol{r_t})\, d\boldsymbol{r'} = \boldsymbol{\mathcal{A}_e}[\boldsymbol{W}(\boldsymbol{r}, \boldsymbol{r_t})]$$

data equation

$$\boldsymbol{W}(\boldsymbol{r}, \boldsymbol{r_t}) = \boldsymbol{\chi}\boldsymbol{E_i}(\boldsymbol{r}, \boldsymbol{r_t}) + \boldsymbol{\chi} \int_{\Omega} G_b(\boldsymbol{r}, \boldsymbol{r'})\, \boldsymbol{W}(\boldsymbol{r'}, \boldsymbol{r_t})\, d\boldsymbol{r'} =$$

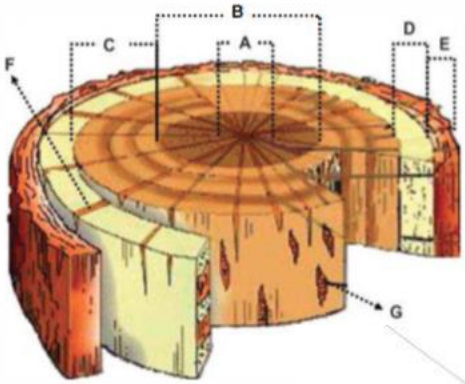
state equation

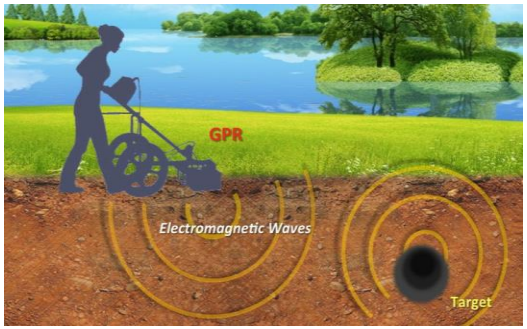
$$= \boldsymbol{\chi}\boldsymbol{E_i}(\boldsymbol{r}, \boldsymbol{r_t}) + \boldsymbol{\chi}\boldsymbol{\mathcal{A}_i}[\boldsymbol{W}(\boldsymbol{r}, \boldsymbol{r_t})]$$

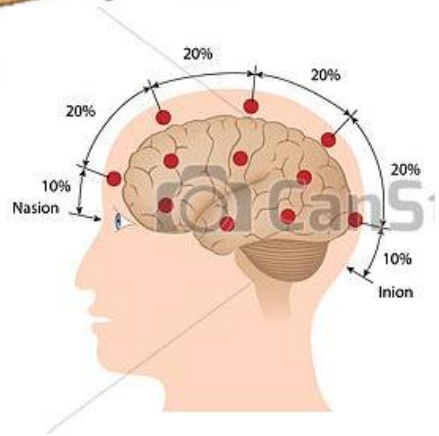
non-linear and ill-posed inverse problem

qualitative imaging method

- **LOCATION AND SHAPE RECONSTRUCTION**
  - ➢ **sufficient in several applications**
  - ➢ **useful a priori information**



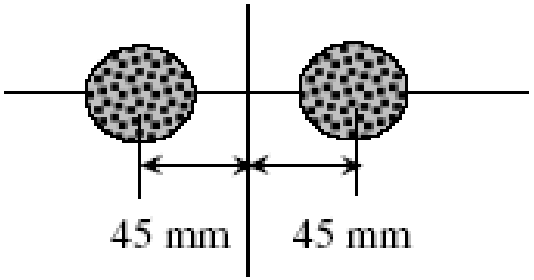


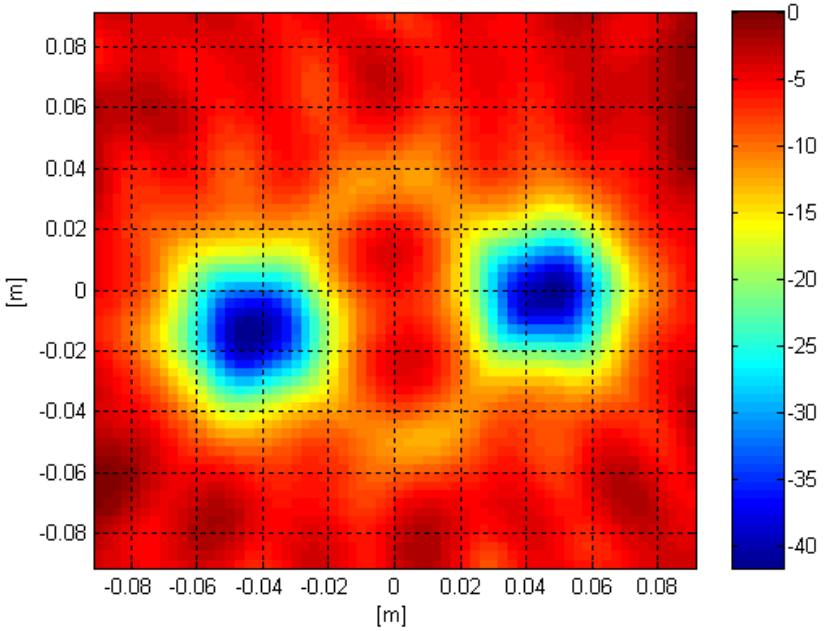


a popular shape indicator: linear sampling method (LSM)

LSM indicator

***TwinDielTM* Fresnel targets**



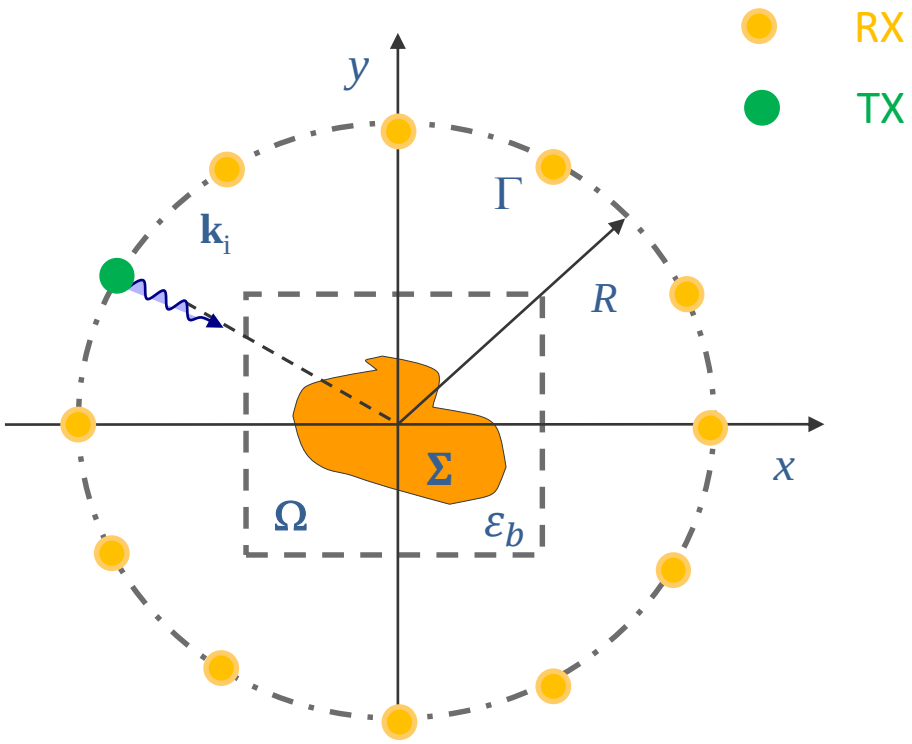


It works only in case of Multiview measurement configuration.

a new qualitative method

- **Use of Equivalence Principles**
- **Solution of a constrained Inverse Source Problem**
- **Regularization by Sparsity Promotion**

scattering experiments



Whatever the performed scattering experiments (Multiview, Multifrequency, etc…), the support of W is exactly equal to Σ.

inverse source for shape reconstruction

inverse *obstacle* problem → **inverse *source* problem**

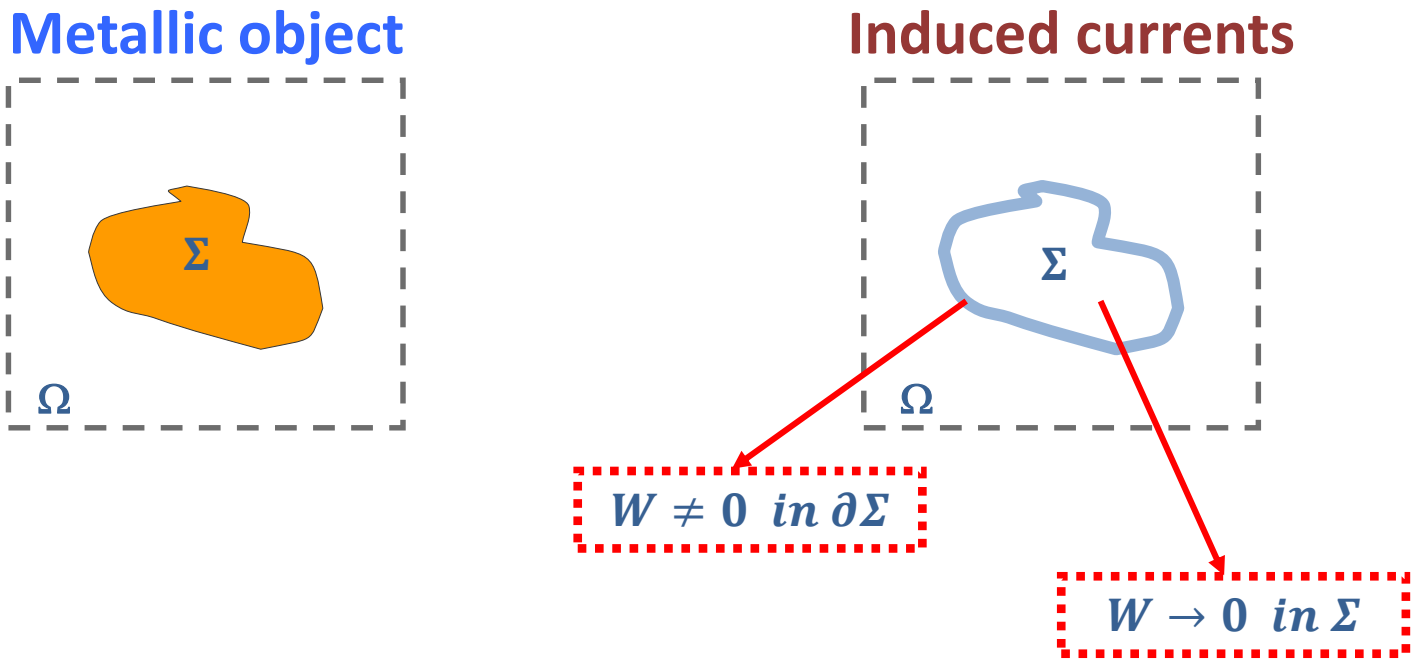
$$\boldsymbol{E_s(r_m, r_t) = \mathcal{A}_e\,[W(r, r_t)]}$$

$$\boldsymbol{supp[W(r, r_t)] = \{r \in \Omega:\ W(r, r_t) \neq 0\ \ \forall r_t\} = \Sigma}$$

Unfortunately, each single inverse source problem is severely ill-posed.

Significant profit can be gained by enforcing some expected properties on W.

an interesting circumstance



The induced currents are localized just in the boundary of the unknown object.

Advantages can be taken from the Compressive Sensing (CS) theory.

inverse source and CS for shape reconstruction

The shape reconstruction problem can be reduced to the well-known '*Basis Pursuit*' problem:

$$\min \|W(\boldsymbol{r}, \boldsymbol{r_t})\|_1$$

$$s.t. \quad \|E_s(\boldsymbol{r_m}, \boldsymbol{r_t}) - \mathcal{A}_e[W(\boldsymbol{r}, \boldsymbol{r_t})]\|_2 \leq \delta$$

inverse source and CS for shape reconstruction

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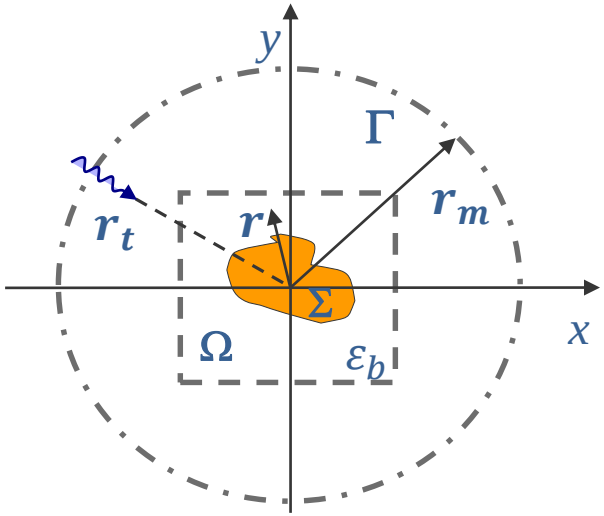
The request for sparseness is <u>still not sufficient</u> to regularize the problem.

Information available from the different experiments can be considered.

inverse source and CS for shape reconstruction

In order to enforce a congruity between the different experiments, an auxiliary variable can be defined as:



$$\mathcal{B}(\boldsymbol{r}) = \max_{\boldsymbol{r_t}} |W(\boldsymbol{r}, \boldsymbol{r_t})|$$

inverse source and CS for shape reconstruction

In order to enforce a congruity between the different experiments, an auxiliary variable can be defined as:

$$\mathcal{B}(\boldsymbol{r}) = \max_{\boldsymbol{r_t}} |W(\boldsymbol{r}, \boldsymbol{r_t})|$$

By enforcing sparsity on $\mathcal{B}$:

Convex problem

$$\min \|\mathcal{B}(\boldsymbol{r})\|_1$$

$$s.t. \quad \|E_s(\boldsymbol{r_m}, \boldsymbol{r_t}) - \mathcal{A}_e[W(\boldsymbol{r}, \boldsymbol{r_t})]\|_2 \leq \delta$$

$$|W(\boldsymbol{r}, \boldsymbol{r_t})| \leq \mathcal{B}(\boldsymbol{r}), \qquad \forall\, \boldsymbol{r_t}$$

inverse source and CS for shape reconstruction*

$$\min \|\mathcal{B}(\boldsymbol{r})\|_1$$

$$s.t. \quad \|E_s(\boldsymbol{r_m}, \boldsymbol{r_t}) - \mathcal{A}_e[W(\boldsymbol{r}, \boldsymbol{r_t})]\|_2 \leq \delta$$

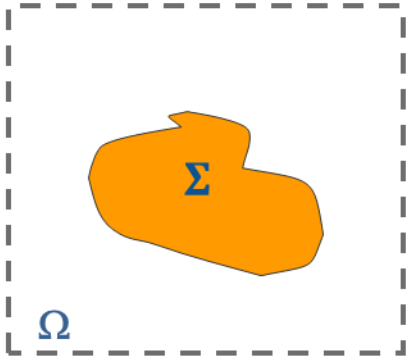
$$|W(\boldsymbol{r}, \boldsymbol{r_t})| \leq \mathcal{B}(\boldsymbol{r}), \qquad \forall\, \boldsymbol{r_t}$$

- $\mathcal{B}$ **enforces sparsity of the currents of each single experiment.**
- $\mathcal{B}$ **enforces a congruity among the different experiments.**
- $\mathcal{B}$ **plays the role of a 'boundary' indicator.**

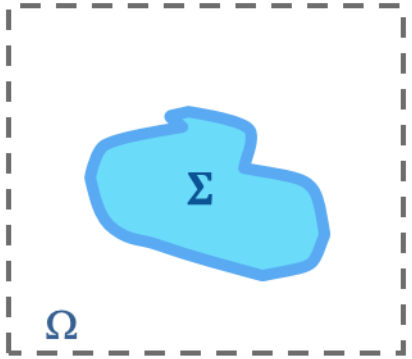
[*] "Shape Reconstruction via Equivalence Principles, Constrained Inverse Source Problems and Sparsity Promotion", PIER, 2017.

what about penetrable targets?

Dielectric object



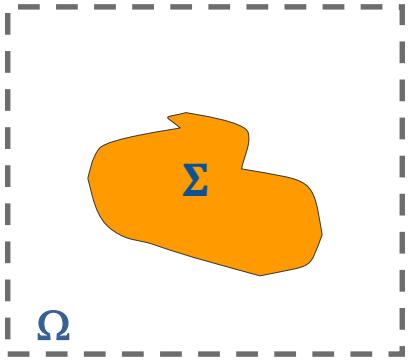
Induced currents



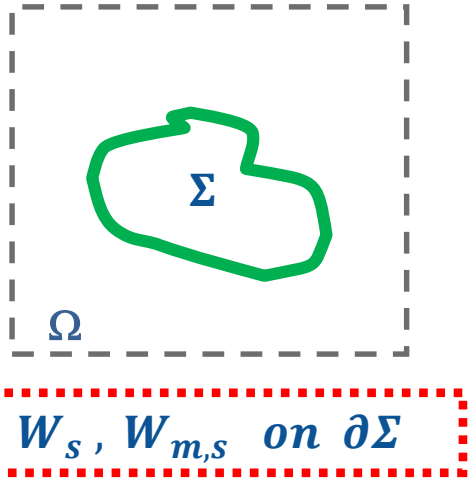
$W \neq 0 \ in \ \Sigma$

what about penetrable targets?

Dielectric object



Equivalent surface currents



By virtue of the equivalence theorem, W can be removed and substituted by equivalent electric W_s and magnetic $W_{m,s}$ surface currents.

inverse source and CS for shape reconstruction*

$$\min \|\mathcal{B}(\boldsymbol{r})\|_1$$

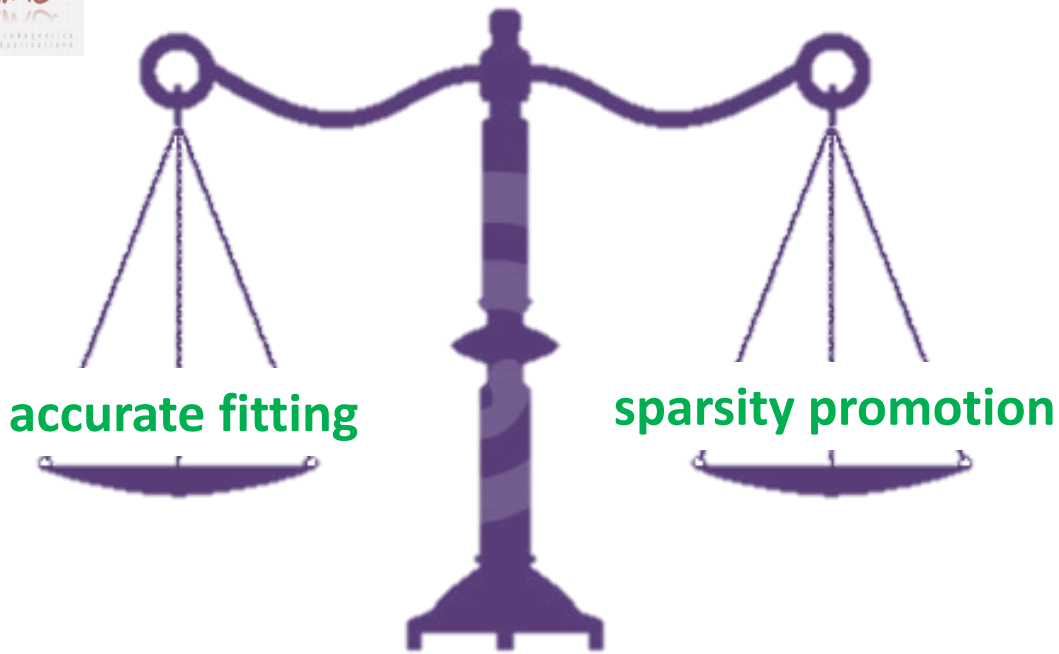
$$s.t. \quad \left\| E_s(\boldsymbol{r_m}, \boldsymbol{r_t}) - \mathcal{A}_e{}^{EE}[W_s(\boldsymbol{r}, \boldsymbol{r_t})] - \boldsymbol{\mathcal{A}}_e{}^{EH}\left[\boldsymbol{W}_{m,s}(\boldsymbol{r}, \boldsymbol{r_t})\right] \right\|_2 \leq \delta$$

$$|W_s(\boldsymbol{r}, \boldsymbol{r_t})| \leq \mathcal{B}(\boldsymbol{r}), \qquad \forall\, \boldsymbol{r_t}$$

$$\frac{\left|\boldsymbol{W}_{m,s}(\boldsymbol{r}, \boldsymbol{r_t})\right|}{\zeta} \leq \mathcal{B}(\boldsymbol{r}), \qquad \forall\, \boldsymbol{r_t}$$

- $\mathcal{B}$ **enforces sparsity of the currents of each single experiment.**
- $\mathcal{B}$**enforces a congruity among the different experiments.**
- $\mathcal{B}$ **plays the role of a 'boundary' indicator.**

[*] "Shape Reconstruction via Equivalence Principles, Constrained Inverse Source Problems and Sparsity Promotion", PIER, 2017.

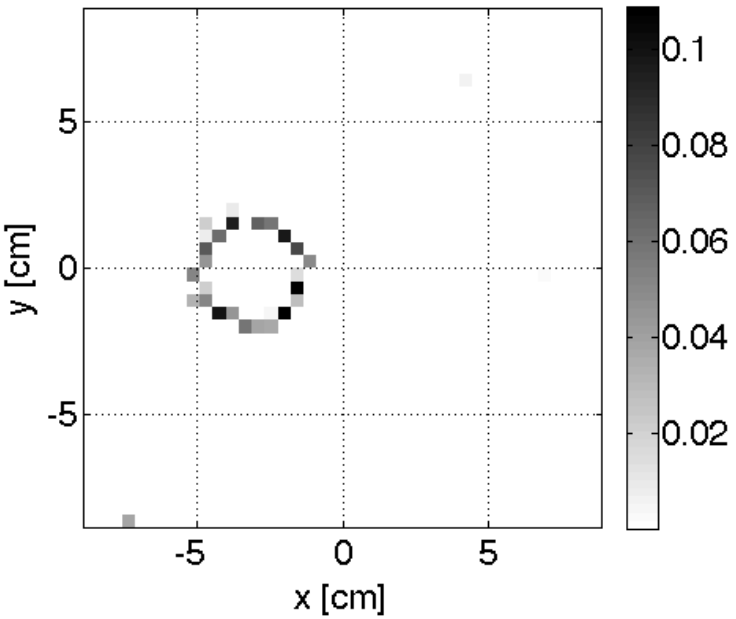


- the sparsity promotion guarantees an estimated support not exceeding the true one.
- the request of accurate fitting allows to get rid of solutions having smaller support.
- the approach is expected to estimate at least their convex hull.

[*] "Shape Reconstruction via Equivalence Principles, Constrained Inverse Source Problems and Sparsity Promotion", PIER, 2017.

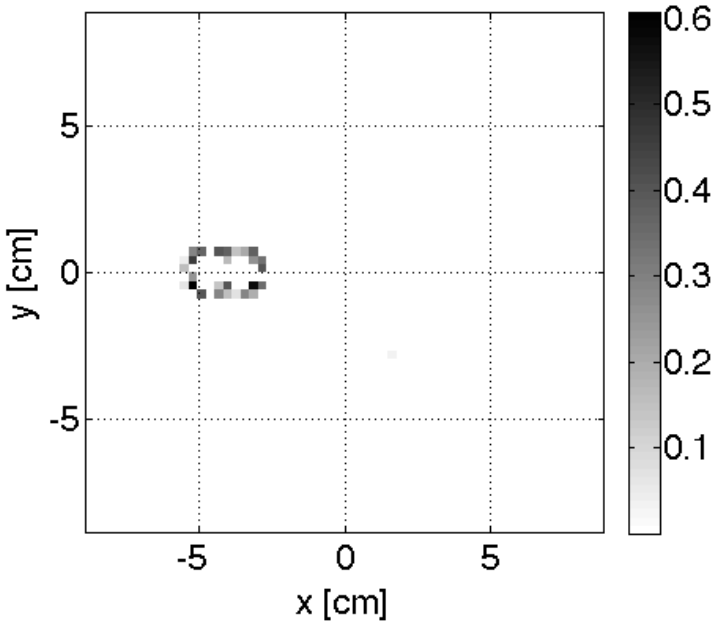
2001 Fresnel data

DielTM target - homogeneous circular dielectric cylinder



f=12 GHz, MxT=18x18, $\delta = 0.5\|E_s\|_2$

RectTM_Dece target - rectangular metallic object

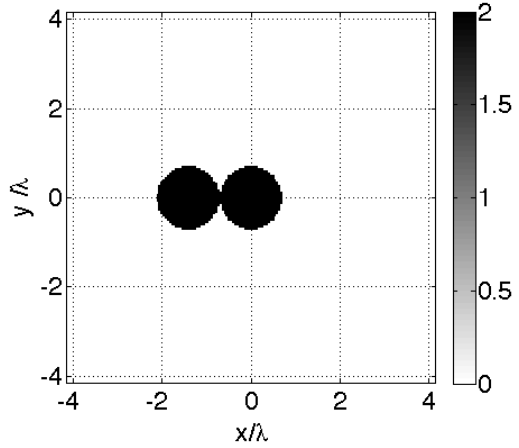


f=16 GHz, MxT=18x18, $\delta = 0.5\|E_s\|_2$

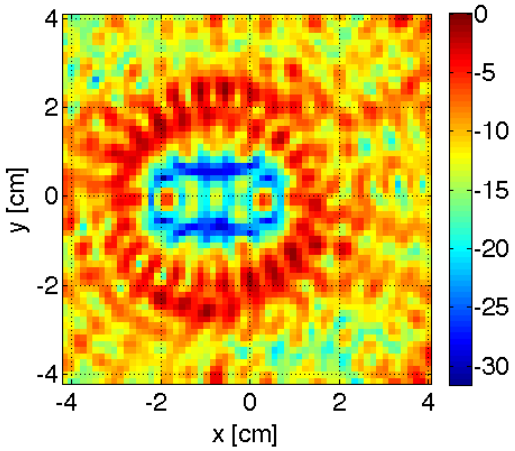
[*] K. Belkebir and M. Saillard, "Special section: Testing inversion algorithms against experimental data," Inverse Problems, 2001.

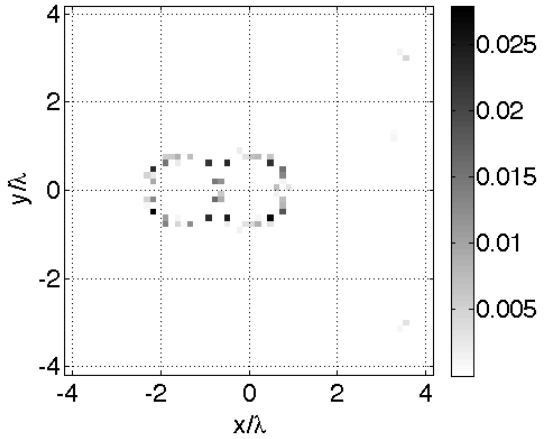
simulated data

non convex dielectric target



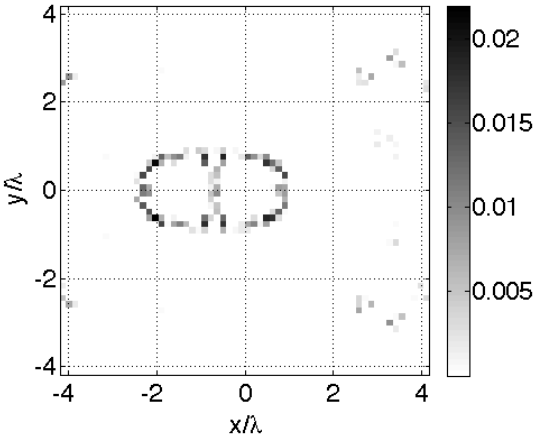
***LSM* indicator**





multiview data

MxT=24x24, f=14GHz

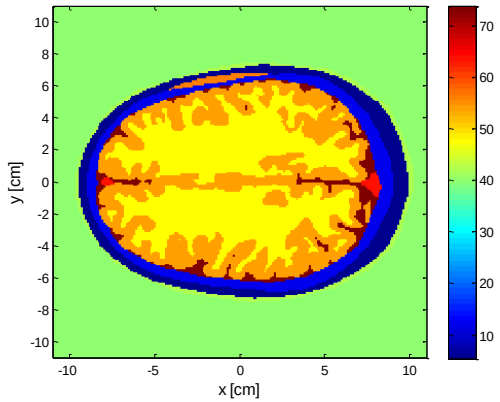


multiview-multifrequency data,

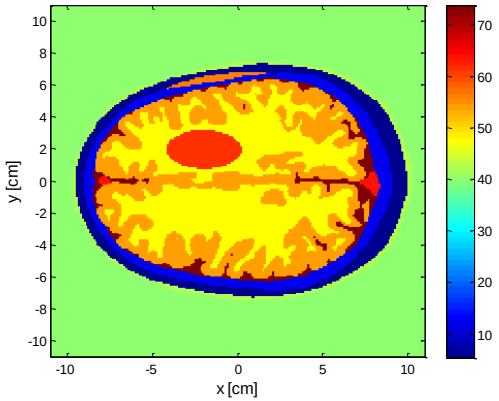
MxT=24x24, f=14-15GHz

differential imaging for brain stroke monitoring*

reference scenario



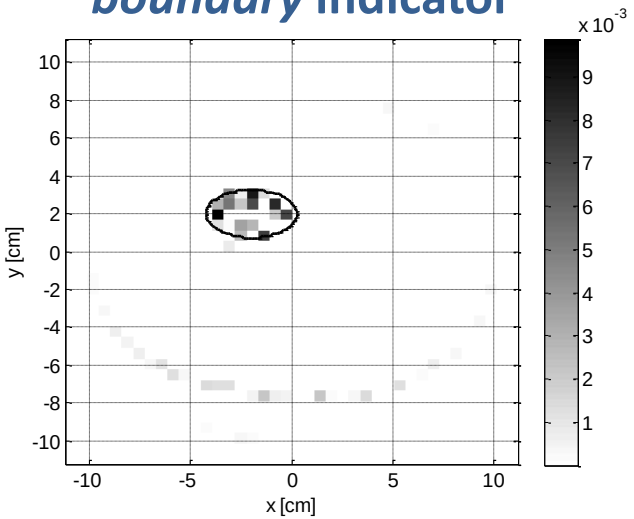
scenario with stroke



E_s

E_{s1}

***boundary* indicator**

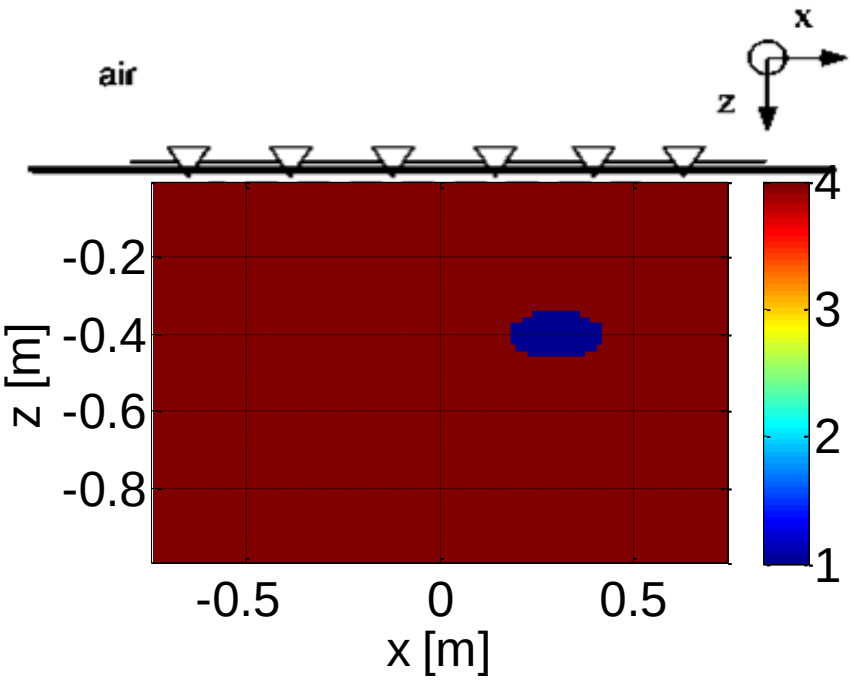


f=1 GHz, MxT=32x16, $\delta = 0.5\|E_s\|_2$

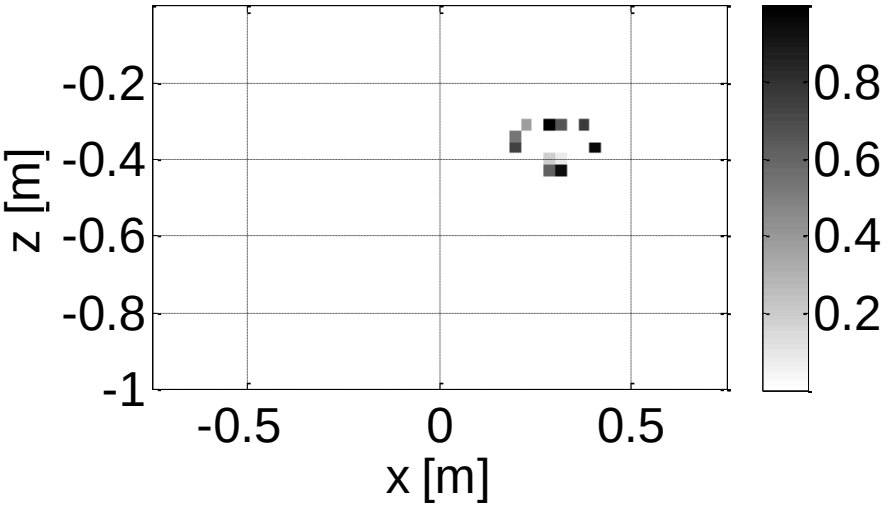
[*] Scapaticci et al., "A feasibility study on microwave imaging for brain stroke monitoring," PIER B, 2012.

imaging of buried object*

metallic buried target



***boundary* indicator**



f=400 MHz, MxT=16x16, $\delta = 0.5\|E_s\|_2$, $L_{array} = 3m$

[*] M. T. Bevacqua, "A Novel Approach for Qualitative Imaging of Buried PEC Scatterers," TELKOMNIKA, 2017.

conclusions

- A new method for estimating the support of both dielectric and metallic targets.
- It is based on equivalence principles, constrained inverse source problems and sparsity promotion.
- Remarkable results with both numerical and Fresnel experimental data.
- Applicability to a huge variety of different measurement configuration, including diversity in angle, in frequency, as well as possible combinations of them.

Thank you for the attention !

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comparisons with other methods

Proposed approach

- linear problem
- for PEC and dielectric targets
- any kind of experiments
- no approximation
- no need of selecting a fixed threshold to discriminate between point inside and outside the scatterer

Linear Sampling method

- linear problem
- for PEC and dielectric targets
- only multiview experiments
- no approximation
- need of selecting a fixed threshold to discriminate between point inside and outside the scatterer

comparisons with other methods

Proposed approach

- linear problem
- for PEC and dielectric targets
- any kind of experiments
- recovering of the overall boundary of the target

Kirchhoff approximation based approach

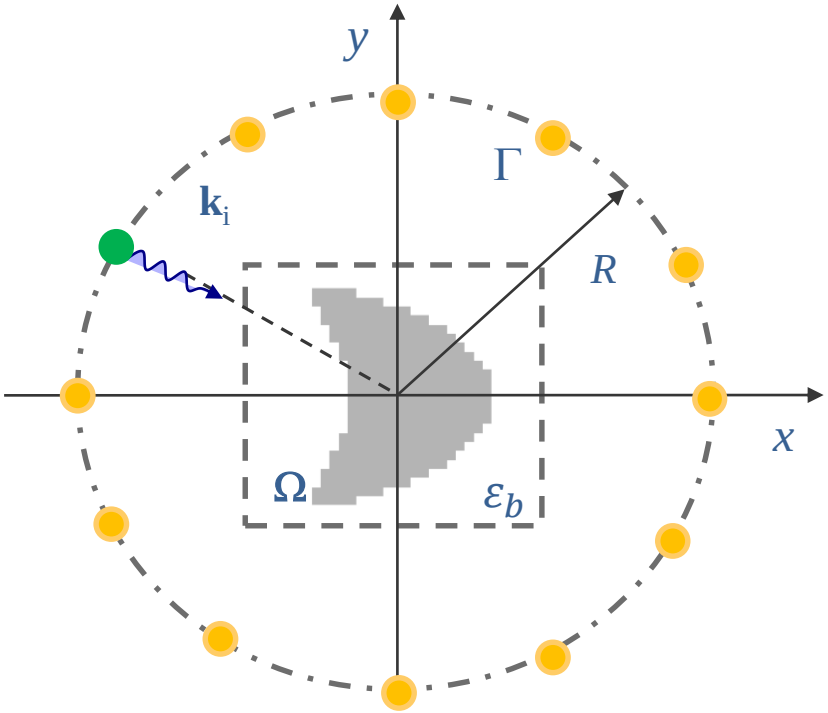
- linear approximation
- just for PEC
- only singleview experiments together with multifrequency experiments
- recovering of just the illuminated side

comparisons with other methods

Proposed approach	Approach in [*]
• Inverse source problem based approach	• introduction of local shape function
• for PEC and dielectric targets	• just for PEC
• deterministic approach	• probabilistic approach
• enforcing of coherence among different experiments	• enforcing of coherence among different experiments
• definition of a unique indicator for the different experiments – no need of posteriori combination	• a posteriori combination of the results came from the different views

[*] Poli et al., "Imaging sparse metallic cylinders through a local shape function Bayesian compressive sensing approach," JOSA A, 2013

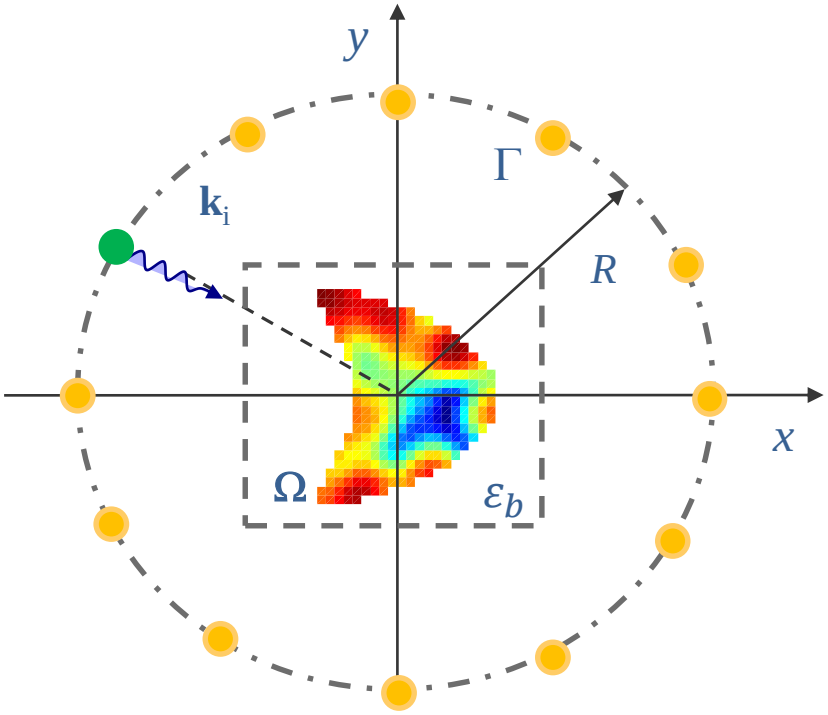
multiview experiments



RX

TX

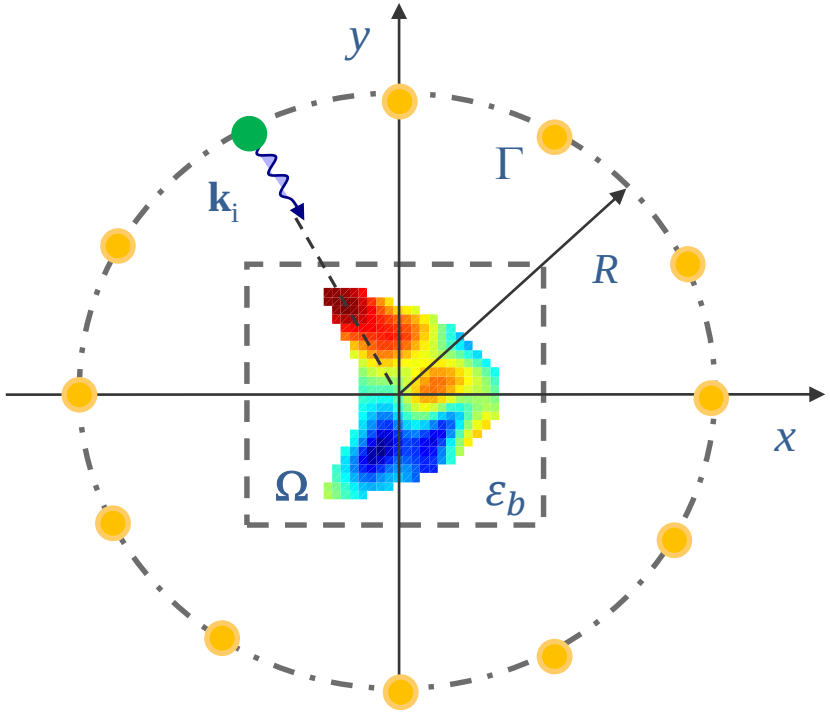
multiview experiments



RX

TX

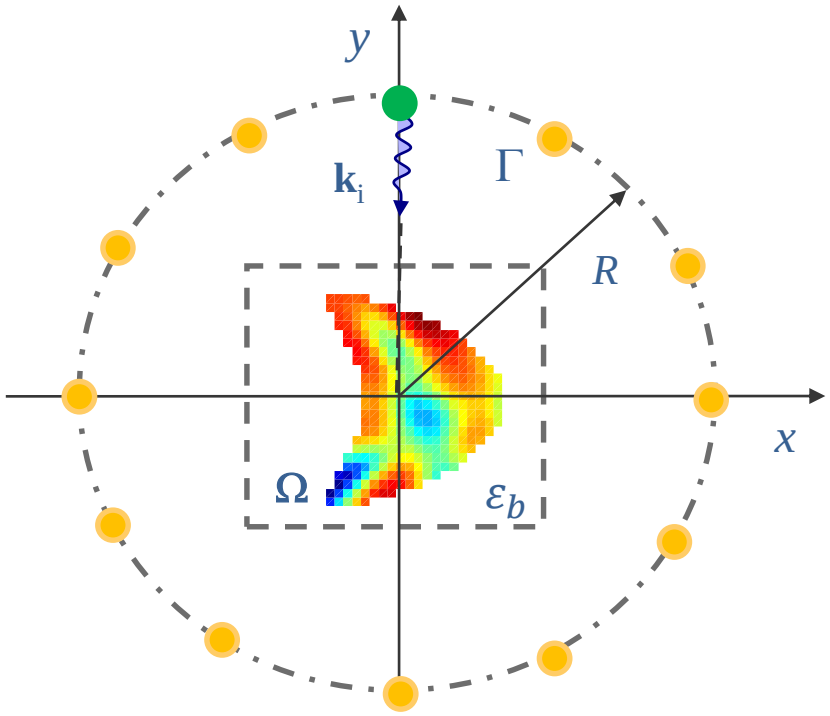
multiview experiments



RX

TX

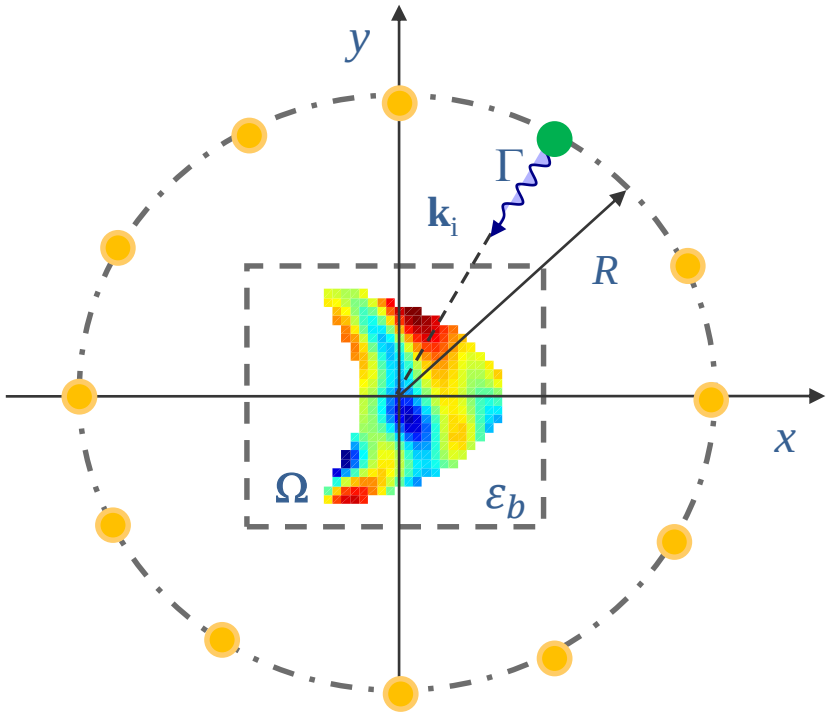
multiview experiments



RX

TX

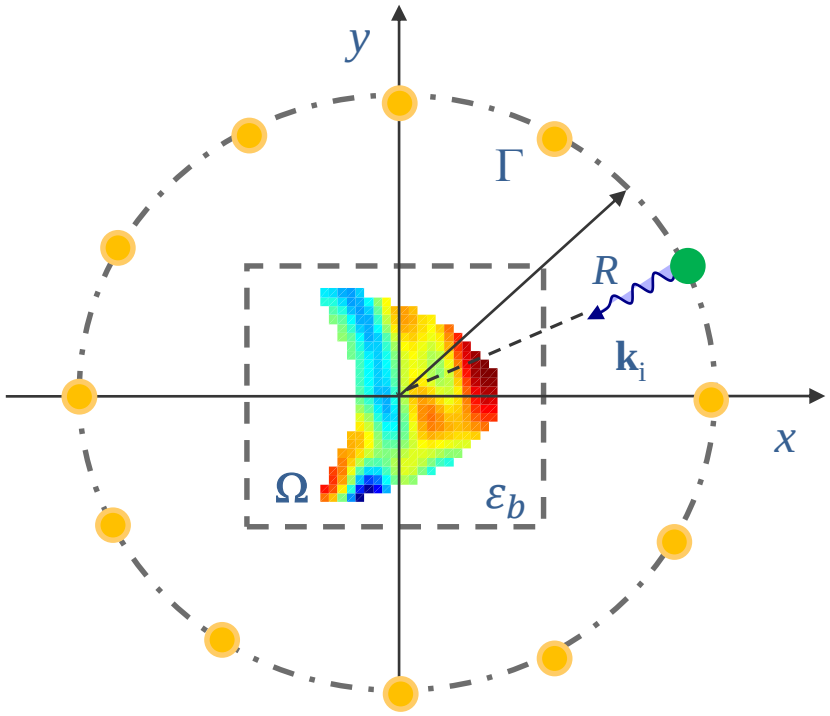
multiview experiments



RX

TX

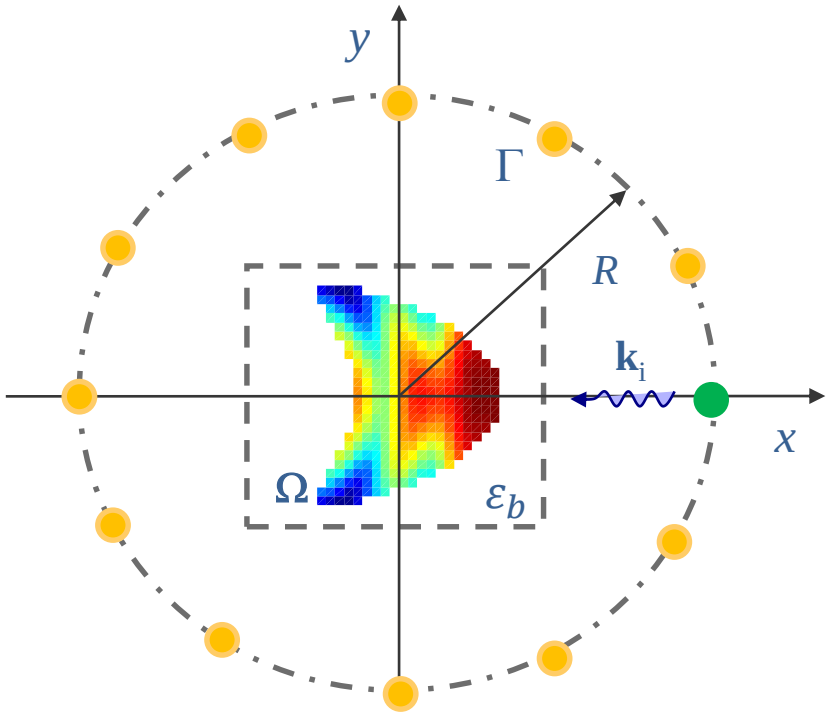
multiview experiments



RX

TX

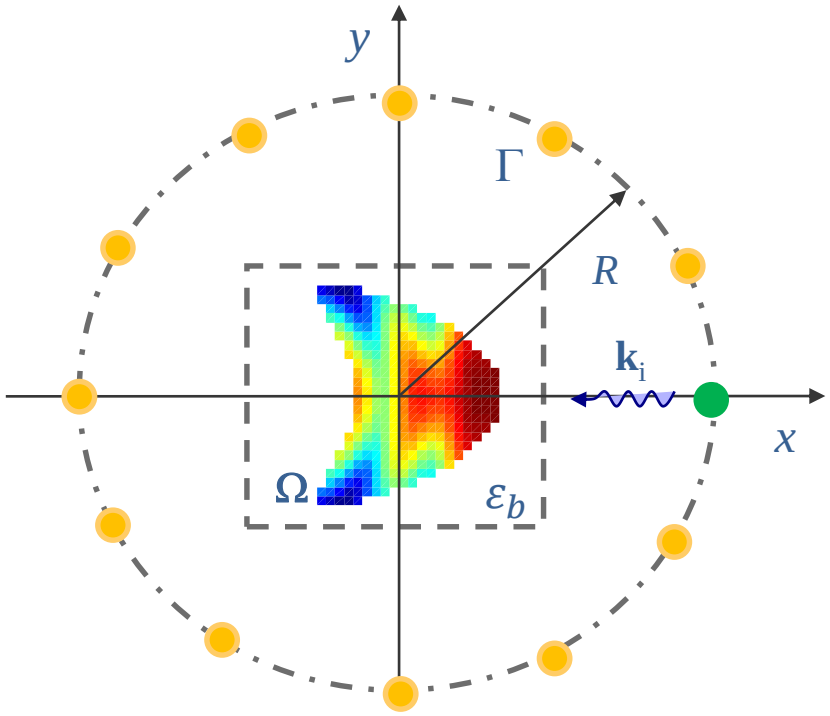
multiview experiments



RX

TX

multiview experiments

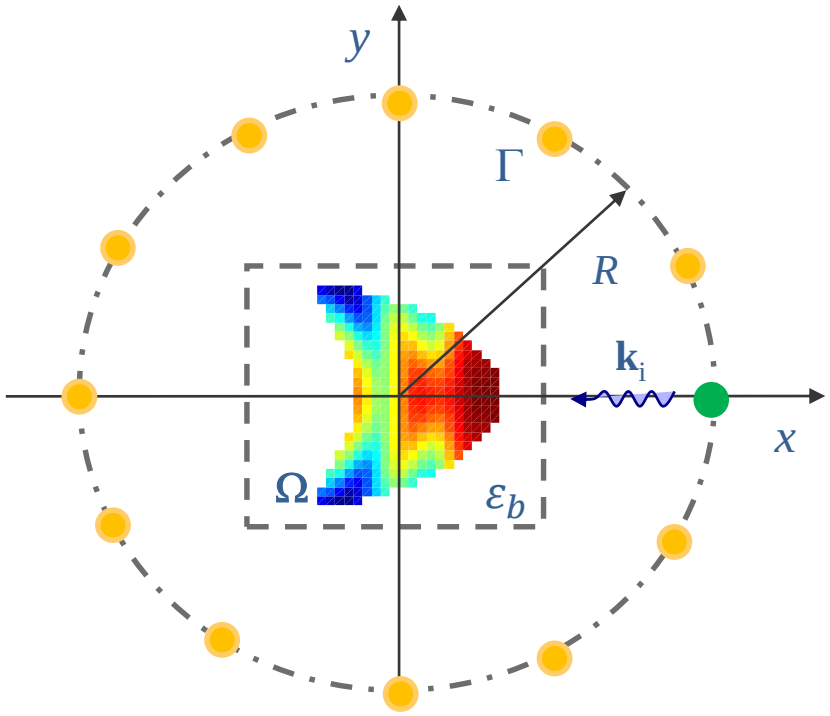


RX

TX

Whatever the incident angles, the support of W is exactly the same of the one of the unknown scatterer.

multiview experiments



RX

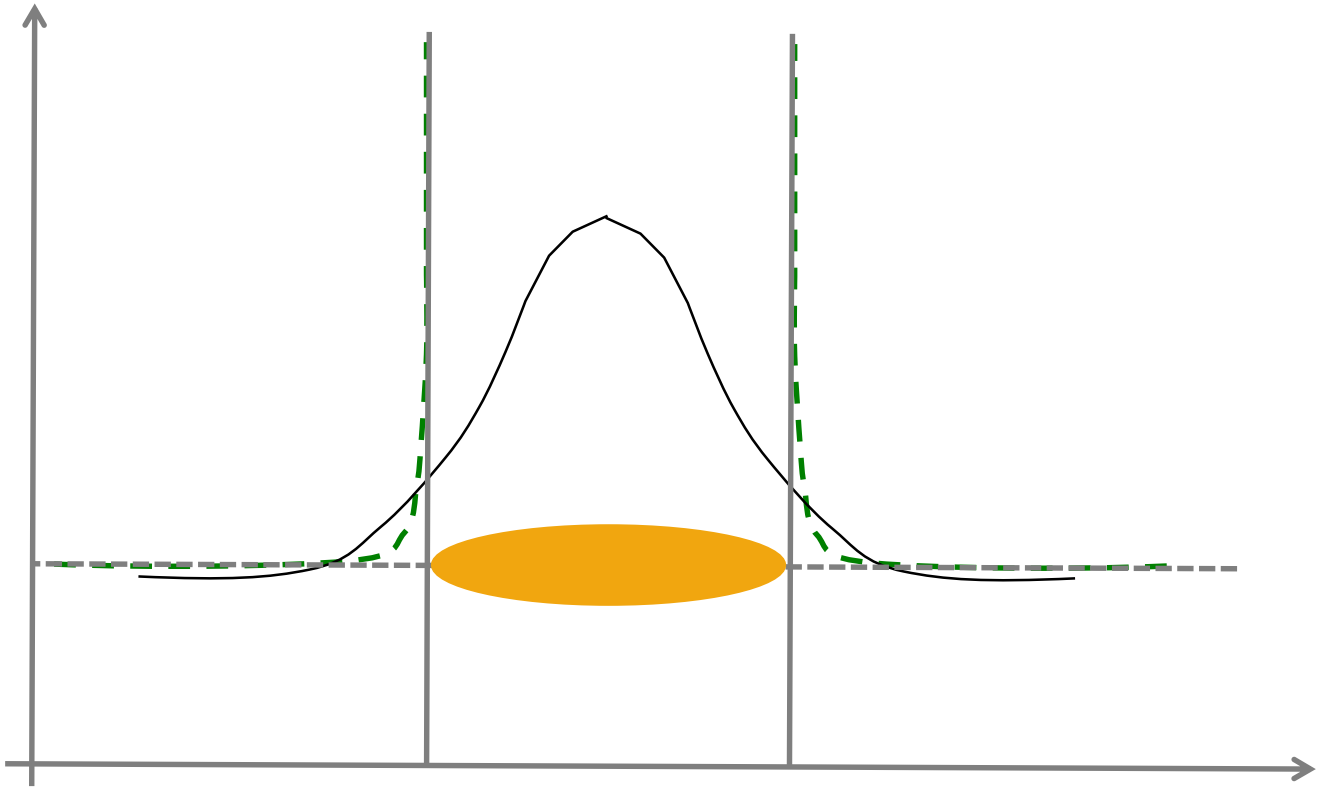
TX

Whatever the performed scattering experiments (Multiview, Multifrequency, etc…), the support of W is exactly the same of the one of the unknown scatterer.

qualitative imaging method

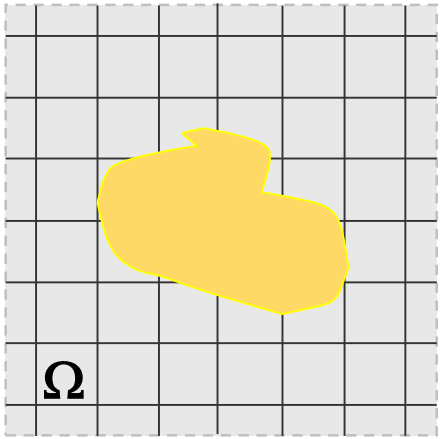
- **LOCATION AND SHAPE RECONSTRUCTION**
  - ➢ **sufficient in several applications**
  - ➢ **useful a priori information**
- **A HIGH FLEXIBILITY**
  - ➢ **dielectric and metallic objects**
  - ➢ **no approximations or a priori information**
- **COMPUTATIONAL EFFICIENCY**
  - ➢ **straightforward implementation**
  - ➢ **“real” time elaborations**

qualitative imaging method



These methods rely on an indicator function whose values determine whether the tested point lies inside or outside the scatterer.

linear sampling method (LSM)

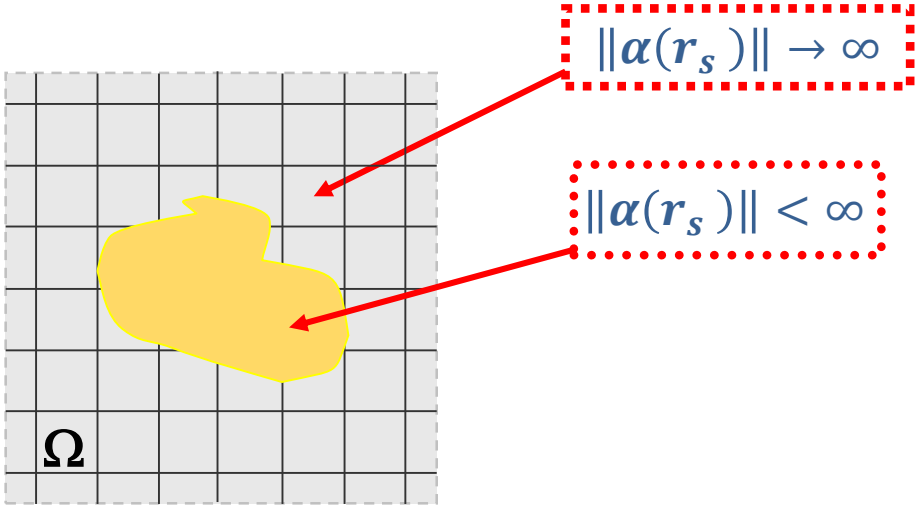


In each point $\boldsymbol{r_s} \in \boldsymbol{\Omega}$, solve the far-field equation

$$\int_{\Gamma} \boldsymbol{E_s}(\boldsymbol{r_m}, \boldsymbol{r_t})\, \boldsymbol{\alpha}(\boldsymbol{r_s}, \boldsymbol{r_t})\, \boldsymbol{d}\, \boldsymbol{r_t} = -\frac{j}{4} H_0^{(2)}(k|\boldsymbol{r_m} - \boldsymbol{r_s}|)$$

$H_0^{(2)}$ represents the field radiated by an elementary source located in $\boldsymbol{r_s}$

linear sampling method (LSM)



$\|\alpha\|$ can be used as a support indicator

Multistatic – Multiview measurement configuration

The "kind" of target is not known (PEC, dielectric, lossy….)

inverse source and CS for shape reconstruction*

- $\mathcal{B}$ **plays the role of a 'boundary' indicator.**

- **trade-off between accurate fitting of the data and the request for a maximally sparse solution.**

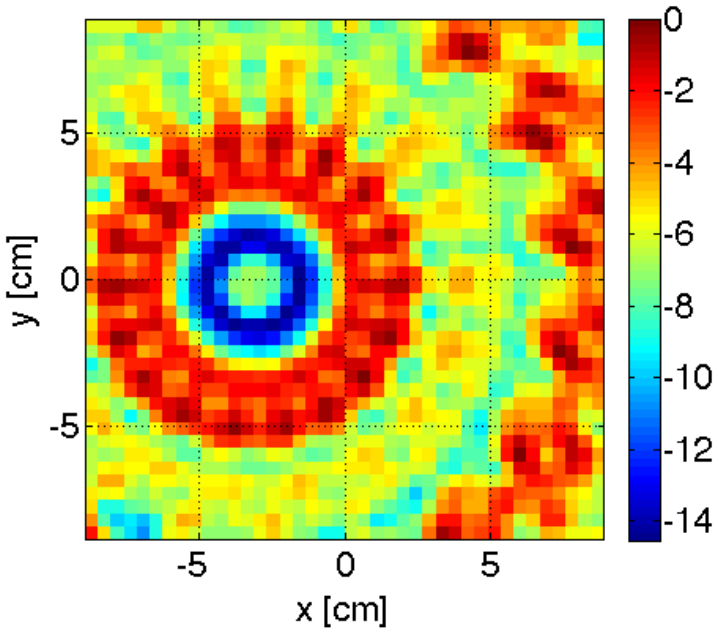
- **easy extension to a huge variety of different measurement configurations.**

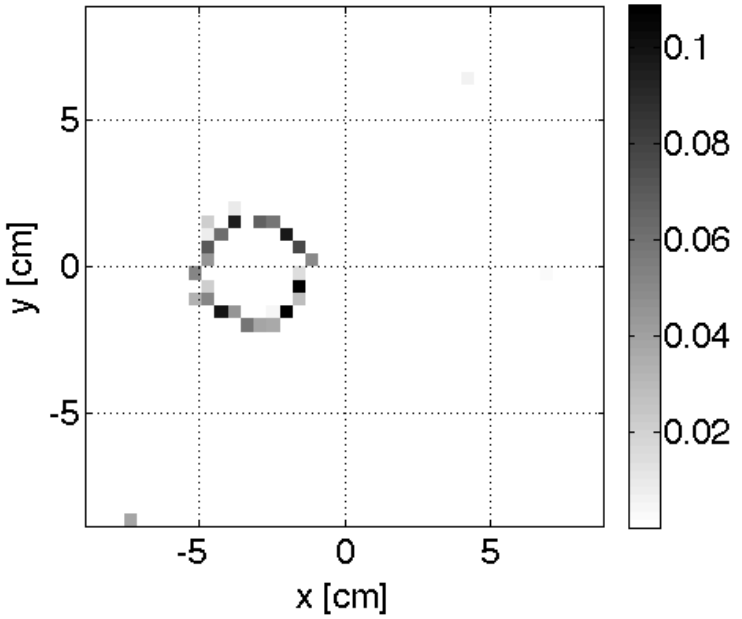
- **not required the knowledge of the incident fields but only some kind of diversity in terms of incident angles and/or frequencies.**

[*] Bevacqua et al., "Shape Reconstruction via Equivalence Principles, Constrained Inverse Source Problems and Sparsity Promotion", PIER, 2017.

2001 Fresnel data

DielTM target - homogeneous circular dielectric cylinder





***LSM* indicator**

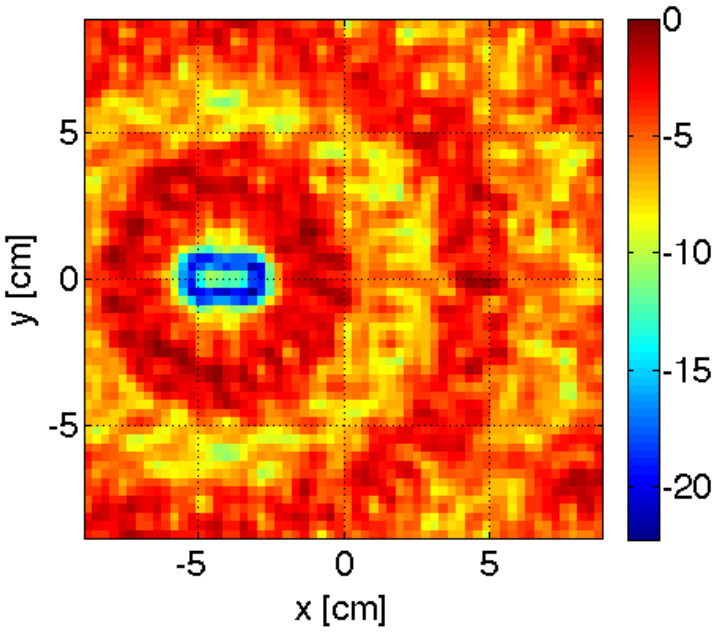
***boundary* indicator**

f=12 GHz, MxT=18x18, $\delta = 0.5\|E_s\|_2$

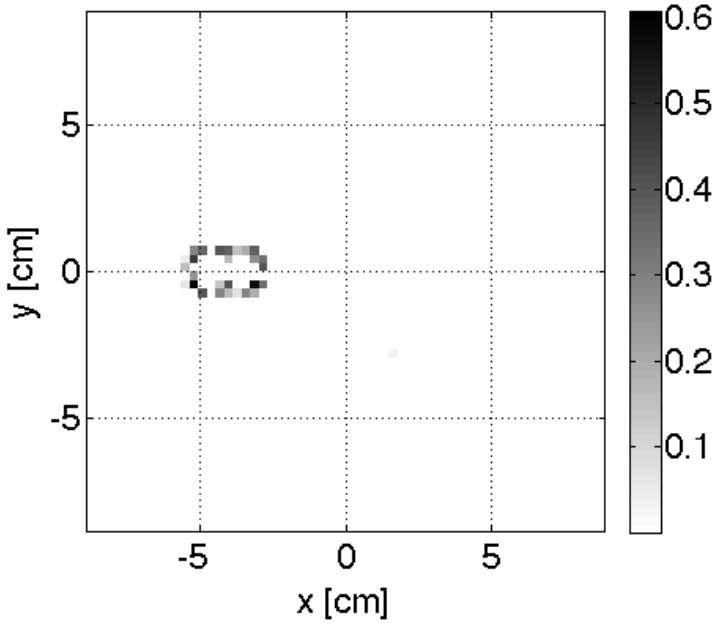
[*] K. Belkebir and M. Saillard, "Special section: Testing inversion algorithms against experimental data," Inverse Problems, 2001.

2001 Fresnel data

***RectTM_Dece* target - rectangular metallic object**



***LSM* indicator**



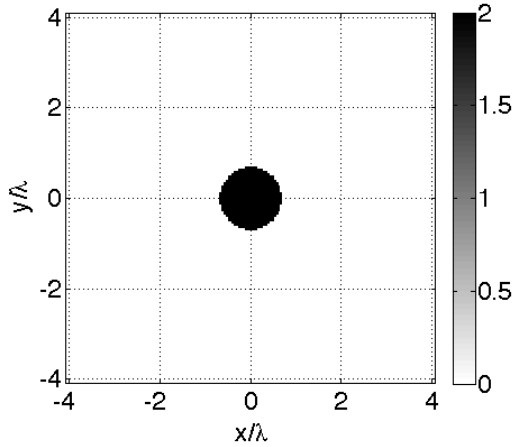
***boundary* indicator**

f=16 GHz, MxT=18x18, $\delta = 0.5\|E_s\|_2$

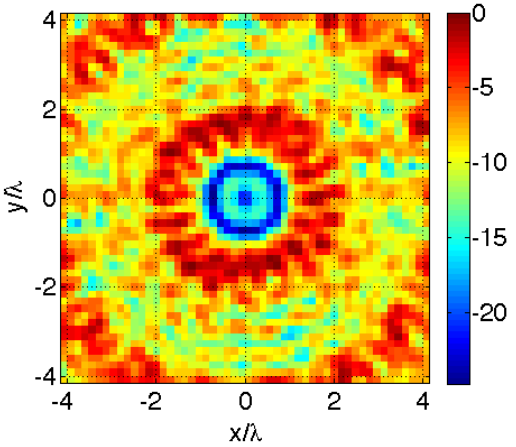
[*] K. Belkebir and M. Saillard, "Special section: Testing inversion algorithms against experimental data," Inverse Problems, 2001.

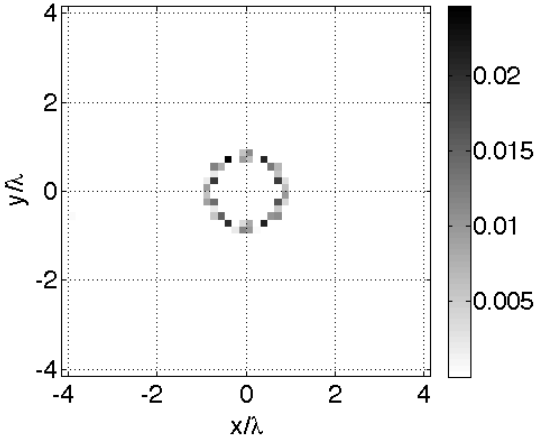
simulated data 1/2

reference profile



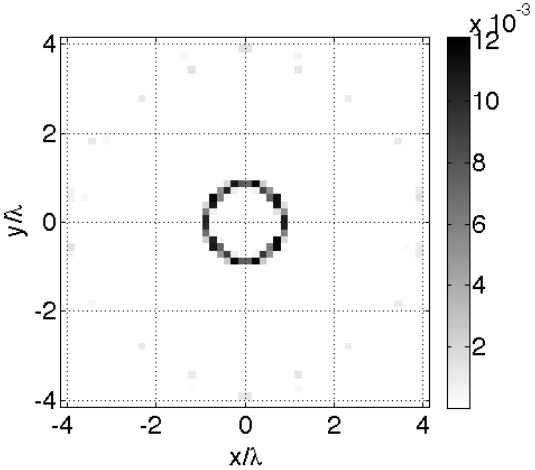
***LSM* indicator**





multiview data

MxT=18x18, f=14GHz

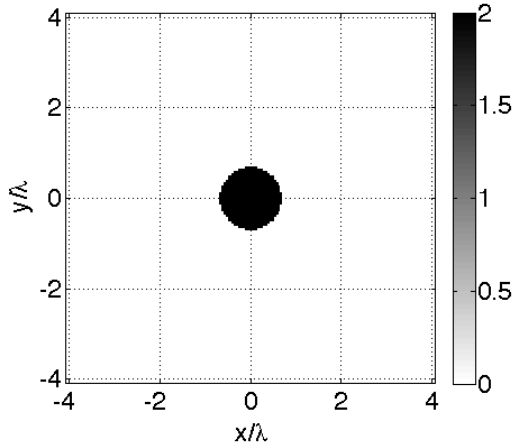


multiview-multifrequency data,

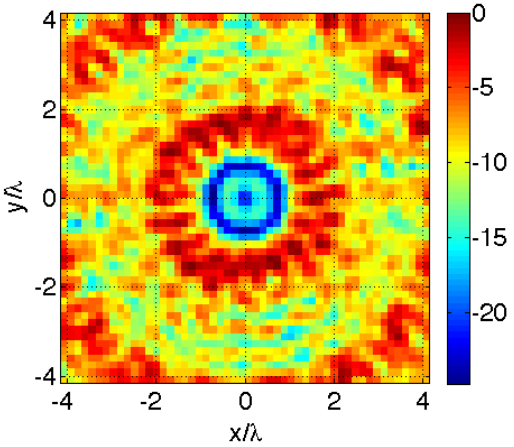
MxT=18x18, f=14-15GHz

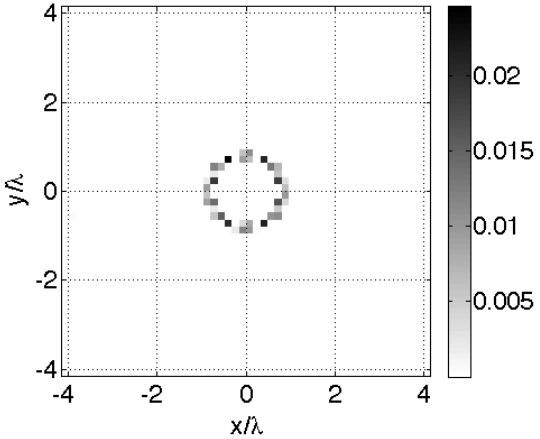
simulated data 1/2

reference profile



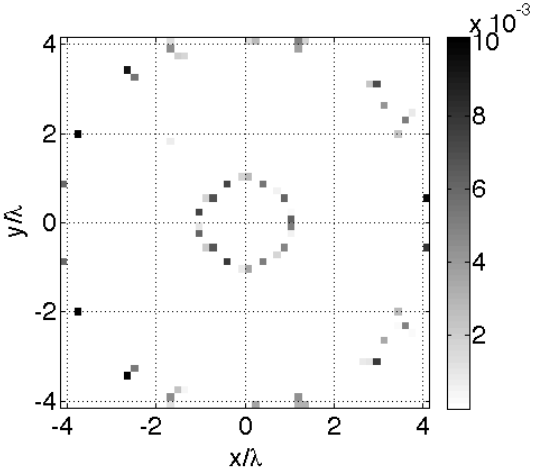
***LSM* indicator**





multiview data

MxT=18x18, f=14GHz



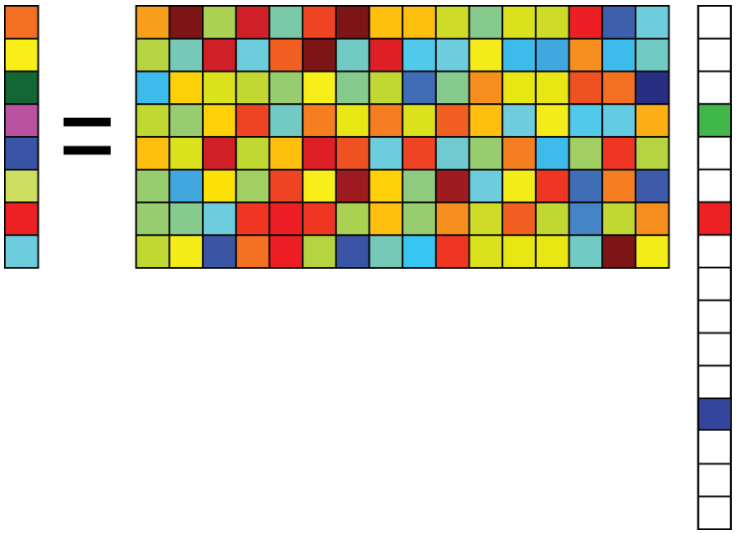
multiview-multifrequency data,

MxT=9x9, f=13-16GHz

compressive sensing theory (CS)

$\mathbf{b} = A\mathbf{x}$

data **unknown**



sparse unknown: can be represented by means of a linear superposition of a few (unknown) elements of the basis $\boldsymbol{\Psi}$.

$$\min\|\mathbf{s}\|_{\ell_1} \; subject\ to \; \|\mathbf{A\Psi s} - \mathbf{b}\| < \delta$$

'BASIS PURSUIT' PROBLEM

compressive sensing theory (CS)

$$\min\|\mathbf{s}\|_{\ell_1} \; subject\ to \; \|\mathbf{A\Psi s} - \mathbf{b}\| < \delta$$

'BASIS PURSUIT' PROBLEM

δ depends on noise on the data, model error and level of required.

In case of no model error, only the noise on data play a role and usually the measurements system specification (SNR) are known.

In case of model error, one can identify the interval to avoid the trivial solution that is:

$$0 < \delta < \|\mathbf{b}\|$$

Then the optimal value is selected as done in cross validation strategy.