Microwave Imaging: from the Virtual Experiments Framework to ECRIS Plasma Diagnostic applications

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Outline

- Introduction to Microwave Imaging and Inverse Scattering
- Main Aspects, Issues and Challenges
- The ‘Virtual Experiments’ Framework (‘well-explored activity’)
- ECRIS Plasma Diagnostics (‘starting/ongoing activity’)
- Perspectives and Conclusions
**Inverse scattering problem: relevant equations (integral formulation)**

External equation (data equation) \[ R \in \Gamma \]

\[
E_v^S(R) = \int_D g(R, r') E_{tot}^v(r') \chi(r') dr' = A_e [E_{tot} \chi]
\]

Internal equation (state equation) \[ r \in D \]

\[
E_{inc} (r) = E_{tot}^v (r) - \int_D g(r, r') E_{tot}^v(r') \chi(r') dr' = (I - A_i [E_{tot} \chi])
\]

Contrast function

\[
\chi(r) = \frac{\varepsilon_S}{\varepsilon_b} - 1
\]

\[
E_v^S = A_e (I - A_i [E_{tot} \chi]^{-1} \chi) = S_{\chi}[\chi]
\]

**Non linear and ill-posed problem**
Inverse scattering problem: solution strategies

- **full-wave (non linear) inversion**

\[
\arg\min \{ \phi(\chi, E_{tot}) + \phi_1 \} \quad \text{(penalty terms from prior knowledge)}
\]

- **approximated inversion models (mostly linear)**

  - Suitable to image large regions
  - Reliable (through regularization)
  - Valid only for the weak scattering regime

  - Linearized models (e.g., Born, Rytov)

**drawbacks**

- Validity of the approximation is limited
- Only qualitative information is achieved (i.e., position, size)

**Minimization procedures can be trapped in local minima!**

**Local search**

**Global search**

**Born approximation**

\[
E^y_s(R) = \int_D g(R, r') E^y_{inc}(r') \chi(r') dr' = A_e [E_{inc} \chi]
\]

Valid only for the weak scattering regime
The key idea of the VE: scattering phenomena are linear (with respect to the incident fields)

\[
\psi_{\text{virt}}(r) = \sum_{\nu=1}^{N} \alpha_{\nu} E_{\text{inc}}^{\nu}(r)
\]

\[
\psi_{\text{tot}}(r) = \sum_{\nu=1}^{N} \alpha_{\nu} E_{\text{tot}}^{\nu}(r)
\]

\[
\psi_{s\text{virt}}(R) = \sum_{\nu=1}^{N} \alpha_{\nu} E_{s}^{\nu}(R)
\]

A design equation for the Virtual Experiments

\[
\arg\min_{\nu} \left\{ \| E_{s}^{\nu}(R) \alpha_{\nu}(r_{s}) - g(|R - r_{s}|) \|^{2} + \lambda \| \alpha_{\nu} \|^{2} \right\}
\]

Sample the domain under test in a grid of sampling points
Inducing circular symmetric scattered field pattern with respect to some ‘pivot points’
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Inducing circular symmetric scattered field pattern with respect to some ‘pivot points’

REMARK: no further measurements are required!
**Solution strategies through the VE**

### Linear approximated model for non weak scatterer

\[
\Psi_s^p(R) = \int_D g(R, r') \Psi_{tot}^p(r') \chi(r') dr' = A_e \Psi_{tot}^p \chi \\
\Psi_{tot}^p(r) = \Psi_{inc}^p(r) + c H_0^{(2)}(k_b |r - r_p|)
\]


### Fast local algebraic solution approach

\[
W^p(r) = \chi \Psi_{tot}^p \approx a_0^p J_0(k_p |r - r_p|) = a_0^p \sum_{k=0}^{n} \frac{1}{n!} \left( \frac{-\chi_p k_b |r - r_p|}{2} \right)^n J_n(k_b |r - r_p|)
\]


### Non linear circular symmetric contrast source promoting search

\[
\arg\min \left\{ \sum_{v=1}^{N} \frac{\| \chi(r) \Psi_{inc}^p(r) + \chi(r) A_i [W^p(r)] - W^p(r) \|^2}{\| \Psi_{inc}^p(r) \|^2} + \frac{\| A_e [W^p(r)] - \Psi_{s}^p(R) \|^2}{\| \Psi_{s}^p(R) \|^2} + \frac{\| \partial W^p(r) / \partial \varphi^p \|^2}{\| \partial \varphi^p \|^2} \right\}
\]


### Distorted iterated Virtual Experiments

\[
\Delta \Psi_{s,k}^p(R) = \int_D \tilde{g}_b(R, r') \Psi_{tot}^p \Delta \chi^k(r') dr' = A_e \Psi_{tot}^p \Delta \chi^k
\]

Experimental benchmark: Fresnel dataset

- **Tx:** $R_v = 720\text{mm} \pm 3\text{mm}$
- **Rx:** $R_m = 760\text{mm} \pm 3\text{mm}$
- **Object rotation range:** $[0^\circ, 360^\circ]$ step=$10^\circ$ (N=36)
- **Rx rotation range:** $[60^\circ, 300^\circ]$ step=$5^\circ$ (M=49)
Experimental benchmark: Fresnel dataset

Non linear approach

Pivot points map

Reconstructed profile
Distorted VE: numerical benchmark

- $N = 24$
- $M = 24$
- Frequency: 5GHz
- Permittivity = 2.2
- Conductivity = 0.1 S/m
- SNR = 25 dB

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<th>$\text{err (k=0)}$</th>
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<th>$\text{RRE}$</th>
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VE reconstruction

Distorted Born reconstruction

VE + Compressive Sensing

First step

Last step
Further possibilities for aspect limited configuration

\[ \alpha_1, \alpha_2, \ldots, \alpha_N \]

\[ \Gamma, \sigma_1, \sigma_x - R_x \]

fictitious measurements
Further possibilities in aspect limited configuration

VE + Compressive Sensing

Born reconstruction

VE reconstruction

SNR=10dB

SNR=30dB

SNR=10dB

Reducing processed data-set

Fictitious measurement setup

[ Virtual Experiments and Compressive Sensing for Subsurface Microwave Tomography (Chapter VIII) ]

In: Compressive Sensing of Earth Observations
Publisher, Edited by C.H. Chen, CRC Press
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Plasma ion sources: Tokamaks vs ECRIS

JET Tokamak for nuclear fusion (energy production)

ECR ION Source: extremely compact plasma machine

- particle density
- temperature
- charge state

\[
\frac{L}{\lambda} \gg 1
\]

\[
\frac{L}{\lambda} < 1
\]
Plasma diagnostic in ECRIS: invasive & non invasive tools

Density measurement technique no-longer based on plasma emission but on “response-on-transmission” of microwaves through the plasma
ECRIS experimental facilities at INFN-LNS Catania

Caesar

Serse

Langmuir probe
Magnet (B = 1 kG)
Vacuum gauge flange
Gas injection
Waveguide port
Optical window flange

Langmuir probe penetration = 200 mm
(total tube = 300 mm)
Classical interferometry for plasmas

How to calculate the density \( n \) of the plasma:

\[
\Delta \varphi = \frac{\omega}{c} \left[ 1 - \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{\frac{1}{2}} \right] L
\]

In plasmas the phase variation depends on the “natural plasma frequency”

\[
\omega_p^2 = \frac{4\pi n e^2}{m e_0}
\]

The plasma frequency depends on the density

Microwave interferometry measures plasma density through a measurement of phase shift.

Drawbacks:

- Suitable for big plasma reactor \( \frac{L}{\lambda} \gg 1 \)
- Mean value of the electron density
Sketch of a Microwave Interferometer

- Conical Horn \( d = 136 \text{ mm} \)
- L = 266 mm

- \(-10\lambda\) - \(-L/2\) - 0 - \(L/2\) - \(10\lambda\)

- Limited ECRIS access probing port
- Multi-paths introduce spurious signals
The presence of plasma (accounted by the plasma frequency $\omega_p$) only shifts the beating frequency, while multipath introduce spurious components in the spectrum.
Sketch of a Microwave Interferometer K-band (18-26.5 GHz) microwave interferometer mounted on the plasma reactor.
Microwave imaging in plasma ECRIS: possible strategies

Inverse profiling

Suitable filtering procedure to depurate the signal form the “multipath” contribution of the cavity

Specific antenna design | High directivity antennas | Non conventional antennas

Use of high frequencies | Time gating

Use of “finite difference” inverse scattering formulation

Study of the imaging performance in metallic cavity

PEC boundary condition | Resolution
Microwave imaging in plasma ECRIS: possible strategies

Inverse profiling

Suitable filtering procedure to depurate the signal form the “multipath” contribution of the cavity

Microwave imaging approach in metallic enclosure

Expected (simulated) plasma profile (line-of-sight)
Microwave Imaging: from the Virtual Experiments’ Framework to ECRIS Plasma Diagnostic applications

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Design of a “frequency swept” interferometer given by the superposition of the reference leg plus the plasma leg waves

The beating frequency can be fixed as long as the ramp relation “freq. vs. time” is chosen in the following way:

$$\omega_{B0} = \frac{\partial \omega}{\partial t} \left( \Delta L \frac{\partial k_g}{\partial \omega} + \frac{2a}{c} \right) = \text{constant}$$

Rapidly increasing frequency produces a beating whose $S(\omega(t))$ shape function assumes the following form:

$$S(\omega) \propto 2A^2 \cos^2 \left\{ \left[ \Delta L \sqrt{\omega^2 - \omega_c^2} + \int_{-a}^{a} \sqrt{\omega^2 - \omega_p^2(l)} \, dl \right] / 2c \right\}$$

The presence of plasma (accounted by the plasma frequency $\omega_p$) only shifts the beating frequency, while multipath introduce spurious components in the spectrum.
Spectral coverage

\[ T = \sum_{i=1}^{\tilde{w}_i^2}, \]  

right singular vectors

Born

Synthetic

Fict. Measurements

void

oil-shale
Sketch of a Microwave Interferometer

Time averaged power of the beating signal

\[ S(\omega) = 2A^2 \cos^2 \left[ \frac{1}{2} \left( \int_L k_{\text{plasma}} \, dl \right) \right] \]

\[ k_{\text{plasma}} = \frac{1}{2} \frac{\omega}{c} \left[ \left( \sqrt{1 - \frac{X}{1-Y}} \right) + \left( \sqrt{1 - \frac{X}{1+Y}} \right) \right] \]
Further possibilities for aspect limited configuration

\[ \alpha_1, \alpha_2, \alpha_N \]

Fictitious measurements

SNR=20dB

400 MHz

VE reconstruction

BA reconstruction

VE + Compressive Sensing

Fictitious measurement setup
Experimental benchmark: inhomogeneous Fresnel dataset

- **Tx**: $R_t = 720\text{mm} \pm 3\text{mm}$
- **Rx**: $R_m = 760\text{mm} \pm 3\text{mm}$
- Object rotation range: $[0^\circ, 360^\circ]$ step=$5^\circ$ ($N=72$)
- Rx rotation range: $[60^\circ, 300^\circ]$ step=$4^\circ$ ($M=61$)
Inverse scattering problem: adopted formulation and configuration

Interrogating field

Scattered field
Inverse scattering problem: solution strategies

- full-wave (non linear) inversion

\[ \text{argmin}\{\phi(\chi, E_{\text{tot}}) + \phi_1\} \] (penalty terms from prior knowledge)

- local search → false solutions may occur
  (initial guess / a priori information / computational burden)

- global search → computational feasibility
  (too large number of unknowns)

Minimization procedures can be trapped in local minima!
Inverse scattering problem: solution strategies

- approximated inversion models (mostly linear)
  
  linearized models (e.g., Born, Rylov)
  - suitable to image large regions
  - reliable (through regularization)

drawbacks

- validity of the approximation is limited
- only qualitative information is achieved (i.e., position, size)

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E^\nu_S(R) = \int_D g(R, r') E^\nu_{inc}(r') \chi(r') dr' = A_e [E_{inc} \chi]
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Valid only for the weak scattering regime

Born approximation