DISCRETIZATION-DIVERSITY SYNTHETIC-FUNCTION TECHNIQUE FOR THE SIMULATION OF LARGE PRINTED ANTENNAS

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Abstract

The Synthetic Function eXpansion (SFX) technique was previously developed and introduced by the authors to reduce the memory storage and solution time of the MoM analysis of printed arrays. The present communication deals with an approach that further enhances the computational properties of the SFX approach by reducing the MoM matrix filling time via a multigrid-like technique.

INTRODUCTION

In many typical arrays the antenna extends over a surface that encompasses several wavelengths, yet subwavelength discretization is to be used for the correct characterization of discontinuities (like edges, recesses, bends, impedance steps, etc.). For such structures, a full-wave analysis can be prohibitively expensive in terms of CPU time and memory occupation, especially if also the beam forming network (BFN) is considered.

In the past, the authors proposed the Synthetic Function eXpansion (SFX) technique [1, and references therein], that permits the analysis of large structures, since both computation time and memory occupation are drastically reduced without affecting the solution accuracy.

The main reason to introduce SFX was the effort to reduce memory storage, thus enabling the treatment of large structures. The SFX technique strongly reduces the linear system solution time, a key factor in large structures where the N^3 complexity of direct solvers is not viable, and where the iterative solvers must face the intrinsic poor conditioning of these problems. SFX did not, per se, reduce the filling time of the MoM matrix: this aspect is addressed here, with a multigrid-like approach. The method can be applied to existing MoM codes, with minimal code modifications.

In order to facilitate the comprehension of the proposed algorithm, we will first briefly summarize the basic ideas of the SFX technique.

THE SYNTHETIC FUNCTION EXPANSION METHOD

The SFX technique is a two-step algorithm. At the first step, the antenna domain (typically a complex structure with BFN and many radiating elements) is decomposed into sub-domains (of arbitrary shape) -termed “blocks”- and on each of them entire-domain basis functions -termed Synthetic Functions (SF)- are generated numerically in the stand-alone configuration. At the second step, these SFs are employed to represent the global solution.

The generation algorithm of the SFs, and their selection criteria, based on the degrees of freedom of the electric field [2], is described in [3].

Assuming identical blocks (to simplify the description), each SF $\underline{\psi}_k$ is a linear combination of the N_f RWG (sub-domain) basis functions $\{\underline{f}_\alpha\}_{1:N_f}$ with support on the mesh of each block, and can be written as:

$$\underline{\psi}_k = \sum_{\alpha=1}^{N_f} \psi_{k\alpha} \underline{f}_\alpha \tag{1}$$

At step 2, the SFs $\underline{\psi}_k$ are used to represent the current on the entire structure, and in view of (1) the block partitioning of the structure means also a block partitioning of the MoM matrix $[Z]$. Diagonal collect the interactions of basis functions $\underline{f}_n$ within the same block, sub-matrices $[Z^{b_1 b_1}]$, while interaction between basis functions in two different blocks b_1, b_2 appear in off-diagonal sub-matrices $[Z^{b_1 b_2}]$.

All basis functions that are not included in any block are grouped together and will form a separate sub-matrix (e.g. those on the BFN, if SFs are considered only on radiators).

Grouping the RWG coefficients of the M SFs of the block b_i in the $N_f \times M$ matrix $[\psi^{b_i}] = [\psi_1^{b_i}, \psi_2^{b_i}, \ldots, \psi_{M_i}^{b_i}]$, the basis change from a given sub-matrix $[Z^{b_1 b_2}]$ of the original MoM matrix to the SF representation is:

$$[Z_{SF}^{b_1 b_2}] = [\psi^{b_1}]^T [Z^{b_1 b_2}] [\psi^{b_2}] \tag{2}$$

Since $M \ll N_f$, the transformation is a compression.

The CPU advantage of the SFX approach derives from the solution of the linear system. Using the SFs in the solution process, the dimension of the resulting impedance matrix is drastically reduced with respect to the initial one. Strongly reducing the dimension of the MoM matrix, the time necessary for the solution of the system will drastically drop down even considering the compression overhead.

Since the computation of the Z_{SF} (as in eq. 2) requires the availability of only the $Z^{b_1 b_2}$ sub-matrices, memory occupation is drastically reduced: when the $[Z_{SF}^{b_1 b_2}]$ sub-matrix is computed it can be stored and the $[Z^{b_1 b_2}]$ sub-matrix can be removed from the memory.

The maximum memory occupation required by the proposed scheme is dictated by the dimension of the largest block, or by the number of the subdomain basis functions not included in the blocks, whichever is largest. Since the memory occupation scales with the square of the dimension of the system, the reduction in memory occupation is very significant.

By this analysis, it results that the proposed method acts both on the reduction of the solution time of the linear system, and on the memory occupation. We address next the issue of reducing also the $[Z_{SF}]$ matrix fill time.

MULTI-GRID MAPPING OF THE SF

The technique is based on the fact that a fine discretization is needed to account for strong, near-field interactions, and hence, to generate the SFs and compute their self-interaction. On the other hand, weaker, far-field interactions between SFs on "distant" blocks are insensitive to the current details of the SFs due to the low pass filtering effect of the Green's function; therefore, a coarser grid would suffice to this aim. The proposed scheme has two grids (extendable to multi-grid), taking advantage of the fact that finer grids are obtained in typical meshers by "h-refinement" of coarser grids, so that both a coarser and (the derived) finer grids are available. The SFs are generated on the fine grid, which is also used to compute the self- and near-couplings of them; when computing the farther interactions, the SFs are mapped onto the coarser grid, and these "coarser SF" are then used to compute their couplings. These non-contacting terms are the most relevant in terms of number of MoM matrix entries.

The resulting scheme will be referred to as Discretization-Diversity SFX (SFX-DD).

On each block we will call $\underline{f}$ the N_f basis functions of the fine mesh, and $\underline{F}$ the N_c basis functions of the coarser mesh, $N_c < N_f$; also, we call $\underline{\Psi}$ the sought-for coarse-grid approximation of $\underline{\psi}$ in (2). On the coarse grid,

$$\underline{\Psi}_k = \sum_{\alpha=1}^{N_c} \Psi_{k\alpha} \underline{F}_\alpha, \qquad \underline{\psi} = \underline{\Psi} + \underline{\varepsilon} \tag{3}$$

and $\underline{\varepsilon}$ is the error of the approximation. We determine the coefficients $\Psi_{k\alpha}$ by requesting that the error $\underline{\varepsilon}$ be zero when tested onto the basis function of the coarser mesh, $\langle \underline{F}_\alpha, \underline{\varepsilon} \rangle = 0, \forall \alpha = 1 : N_c$. This results in the following equation for $[\Psi_k]$:

$$[\Psi_k] = [\Pi^c]^{-1} [\Pi^{cf}] [\psi_k] \tag{4}$$

where $[\Pi^{cf}]$: $\Pi^{cf}_{\alpha\beta} = \langle \underline{F}_\alpha, \underline{f}_\beta \rangle$ and $[\Pi^c]$: $\Pi^c_{\alpha\beta} = \langle \underline{F}_\alpha, \underline{F}_\beta \rangle$ are the projection matrices between the basis functions of the coarser and fine bases.

The procedure can be iterated by further reducing the mesh cell size as the distance between the blocks increases.

NUMERICAL EXAMPLE

The described method was applied to the analysis of a 4×2 array of stacked patches with monolithic feed shown in the inset in Fig. 1. This antenna was chosen because it has a complex structure and two resonant frequencies (in the 1.6 - 1.9 GHz frequency band), yet it can be discretized with a number of unknowns that still affords a standard MoM solution, so as to allow for accuracy checks of the proposed methods. The following analysis is concentrated on the input reflection coefficient, much more critical than other parameters.

At the finest level, as required by conventional MoM, the antenna is discretized by 2729 subdomain basis functions; it was divided in 5 blocks: 4 identical, each of them containing one of the four arms, and one containing the remaining BFN. The 4 identical blocks have support on two layers containing 2 pairs of stacked patches and part of the access lines (see Fig. 2). Each of this block contains $N_f = 618$ basis functions. On the 4 blocks, a second mesh was generated, with $N_c = 198$ basis functions. In this case $N_f/N_c \simeq 3$.

In this example, on each of the 4 identical blocks, 2 natural and 4 coupling SFs were used [3], i.e. $M = 6$. On the BFN the initial discretization was maintained, and no SFs was used.

In Fig. 1 the antenna input reflection coefficient obtained by the SFX-DD approach is compared with the result of the standard MoM solution and with the SFX approach. While simple grid SFX yields results virtually indistinguishable from the reference, a slight deterioration is due to the price paid for the faster SFX-DD approach. Note however, that resonance frequencies are not altered.

In this simple example, the speed-up factor (defined as the ratio of the number of the matrix entries to be computed using standard SFX of MoM and SFX-DD approach) is about 3. For larger structures, where off-diagonal blocks dominate the entry count, the speed-up factor tends to $(N_f/\overline{N}_c)^2$, where $\overline{N}_c$ is the average number of basis function on the coarser mesh.

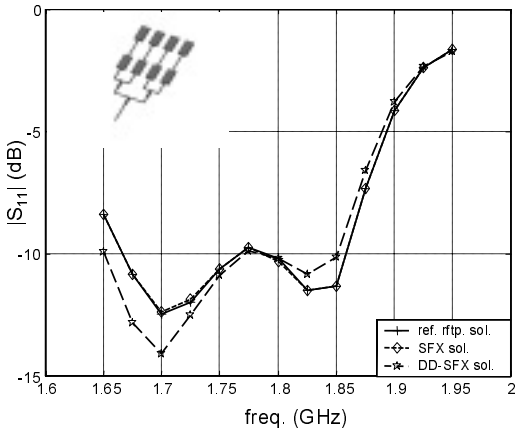

Figure 1: Comparison between different solution schemes: Inset: the geometry of the 4 × 2 array

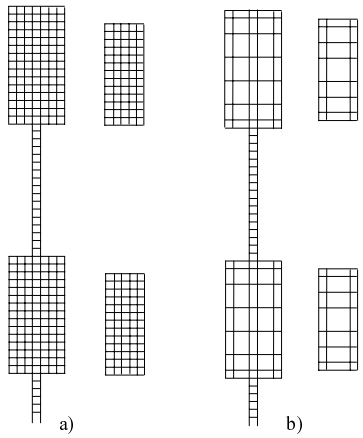

Figure 2: Discretization-Diversity: a) Fine mesh b) Coarse mesh

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