A BMIA/AIM FORMULATION FOR THE ANALYSIS OF LARGE STACKED PATCH ANTENNAS

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Abstract

An efficient technique based on the extension of the Banded Matrix Iterative Approach (BMIA) to a not canonical grid by using the Adaptive Integral Method (AIM) is presented for the analysis of stacked patch antennas of large dimensions. The patches can have arbitrary shape and orientation and are modeled by means of triangular elements.

INTRODUCTION

The analysis of large-scale complex patch antennas by using method of moments (MoM) usually requires large computational resources in terms both of dynamic memory and computation time. Especially for an optimization process, where many slightly different structures have to be analysed, availability of an efficient numerical method is desirable. Similar problems arise in a Monte–Carlo analysis of the scattering from large rough surfaces. In this kind of study the scattered field intensity is averaged over more of one hundred of typical realizations of the assumed scenery built in conformity with its statistical behavior. Obviously, it is very important to minimize the computation time for the analysis of each realization and several techniques have been developed for the fast computation of deterministic profile scattering. These techniques are based on the modification of the classical method of moments (MoM) to allow a fast evaluation of the reaction integral and, when an iterative solver is used, a fast matrix–vector multiplication. Examples include the adaptive integral method (AIM) [1], the fast multipole method (FMM) [2], the matrix decomposition algorithm (MDA) [3], and the banded matrix iterative approach/canonical grid method (BMIA/CAG) [4]. Specifically, the BMIA/CAG method is one of the most efficient when applied to the studying of rough surfaces.

The aim of this paper is to show that the last method, when opportunely modified, can be profitably employed for the analysis of large-scale stacked patch antennas. In particular, the necessity to describe patches of arbitrary shape and size in a regular grid is considered by introducing a set of auxiliary basis functions based on a multipole expansion as in the AIM [1]. Furthermore, special attention has been addressed to the presence of a ground plane, placed below the patches, which reduces the number of terms we need to evaluate.

In order to simplify the formulation, the antennas analysed in the following are supposed characterized by a substrate of dielectric relative permittivity very close to unity, similar to those usually employed by GSM systems having space and frequency diversity and/or in a GPS system mounted on satellites. This hypothesis allows the analysis of patches in free space by using in the procedure an expansion of the free space Green function. The method, however, can be easily extended to dielectric stratified structure with a little effort when discrete complex image method (DCIM) is employed [5], at least when substrates having low relative permittivity are employed.

FORMULATION

The general structure sketched in Fig. 1 can be considered a quasi–planar metallic structure located over an infinite ground plane having a height profile $z = f(x, y)$.

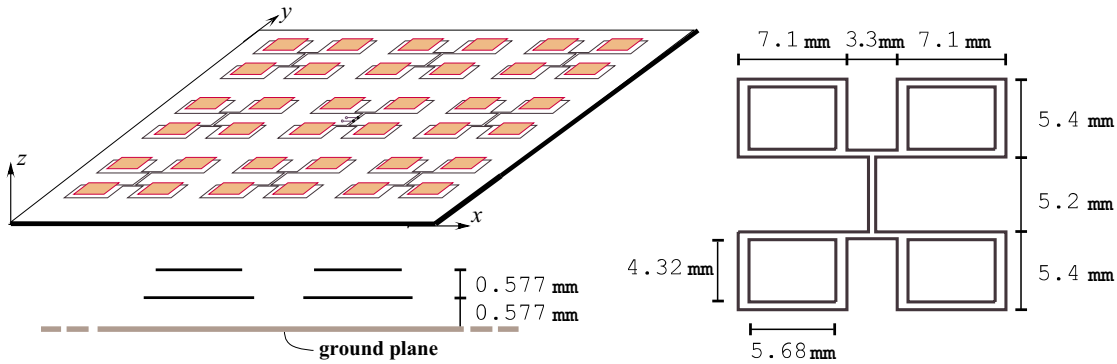


Fig. 1 – Geometry of the problem.

Let $\vec{J}_s$ be the surface current density induced on it by an incident field $\vec{E}^i$ and $\vec{E}^s$ the electric scattered field. The latter can be computed from the surface currents by $\vec{E}^s = -j\omega\vec{A} - \nabla\phi$, with the magnetic vector potential $\vec{A}$ and the scalar potential ϕ defined as

$$\begin{bmatrix} \vec{A}(\vec{r}) \\ \phi(\vec{r}) \end{bmatrix} = \frac{j}{4\pi} \iint_S \begin{bmatrix} -j\mu\vec{J}_s(\vec{r}\,') \\ \nabla \cdot \vec{J}_s(\vec{r}\,')/\omega\varepsilon \end{bmatrix} \cdot G(\vec{r}, \vec{r}\,')\, dS' \,, \tag{1}$$

where $G(\vec{r}, \vec{r}\,') = \exp(-jkR)/R$, and $R = |\vec{r} - \vec{r}\,'|$. By enforcing the boundary condition $\widehat{n} \times \left(\vec{E}^i + \vec{E}^s\right) = 0$ on S, we can derive an integrodifferential equation for $\vec{J}_s$ obtaining $\widehat{n} \times \vec{E}^i\Big|_S = \widehat{n} \times \left(j\omega\vec{A} + \nabla\phi\right)\Big|_S$. The last expression together with equation (1) represent the so–called electric field integral equation (EFIE). For numerical solution of the pertinent integral equation the surface S is discretized into small triangular patches, and the unknown current $\vec{J}_s$ is expanded using a suitable set of basis functions $\vec{j}_n(\vec{r})$ (e.g., Rao–Wilton–Glisson (RWG) basis functions defined by Rao et al. [6]). That is $\vec{J}_s(\vec{r}) = \sum_{n=1}^{N} I_n \vec{j}_n(\vec{r})$, where I_n are unknown coefficients. Then, a method of moments is applied to obtain a linear system of equations, $\mathbb{Z}\,\mathbf{I} = \mathbf{V}$. In classical BMIA the computational efficiency is achieved defining a distance r_d which separates two regions: a near–interaction region and a weak–interaction region. Then, one can write the matrix $\mathbb{Z}$ as superimposition of a strong matrix $\mathbb{Z}^s$ and a weak–interaction matrix $\mathbb{Z}^w$ (i.e. $\mathbb{Z} = \mathbb{Z}^s + \mathbb{Z}^w$). The elements of $\mathbb{Z}^s$ are related to weighting and base functions having distances $d_{xy} = \sqrt{(x - x')^2 + (y - y')^2} < r_d$. In the weak–interaction region we consider $h = |z - z'| = |f(x, y) - f(x', y')| \ll d_{xy}$. So we can approximate the Green's function by using $\widetilde{M}$ terms of the relevant Taylor series with respect to the height h, resulting

$$G(d_{xy}, h) \simeq \sum_{m=0}^{\tilde{M}} a_m(d_{xy}) \exp(-jkd_{xy})\, h^{2m} \,, \tag{2}$$

where $a_0 = 1/d_{xy}$, $a_1 = -(1 + jkd_{xy})/2d_{xy}^3$, $a_2 = (3 + 3jkd_{xy} - k^2 d_{xy}^2)/8d_{xy}^5$, ... As a consequence, the weak–term of the eq. (1) can be written as

$$\sum_{m=0}^{\tilde{M}} \frac{j}{4\pi} \iint_S \begin{bmatrix} -j\mu\vec{J}_s(x', y', z') \\ \nabla \cdot \vec{J}_s(x', y', z')/\omega\varepsilon \end{bmatrix} a_m(d_{xy}) \exp(-jkd_{xy}) \left[z - z'\right]^{2m} dS' \,. \tag{3}$$

Therefore, by assuming the Galerkin discretization scheme, each element of the weak–interaction matrix $\mathbb{Z}^w$ can be expressed as linear combination of four elements of the form

$$Z^{\alpha}_{i,j} = \sum_{m=0}^{M} \iint_S R_m(z)\psi^{\alpha}_i(x,y,z) \iint_S A_m(x-x',y-y')Q_m(z')\psi^{\alpha}_j(x',y',z')\, dS'\, dS\,. \quad (4)$$

where $\alpha = x, y, z, \phi$, ψ^{α}_i is the pertinent base or weighting function and $M = (\widetilde{M}+1)^2 - 1$.

At this point, it is convenient to replace the original basis and weighting current distribution multiplied by $P_m(z) = Q_m(z)$ or $P_m(z) = R_m(z)$ with an approximately equivalent set of pointlike currents. The two current distributions are equivalent in the sense that they generate almost identical fields in the weak–interaction region. We choose the pointlike current elements located at nodes of a regular Cartesian two–dimensional grid, parallel to the x, y plane. Both basis and weighting functions are approximated as linear combination of Dirac delta functions, thus

$$P_m(z)\psi^{\alpha}_n(x,y,z) \simeq \sum_{i=0}^{L} \Lambda^{\alpha,m}_{n,i}\, \delta(x-x_i)\delta(y-y_i)\delta(z-z_c)\,, \quad (5)$$

with $(x_i, y_i) \in C_n$, where C_n is the set of $L+1$ grid nodes closest to the center (x_c, y_c, z_c) of the basis or weghting function support.

However, differently from the classical AIM, since we require that pointlike current elements are bound only to be belong to a regular Cartesian two–dimensional grid having N_g nodes, we enforce the equality

$$\iint_S P_m(z)\psi^{\alpha}_n(x,y,z)(x-x_c)^{m_1}(y-y_c)^{m_2} dS = \sum_{i=0}^{L} \Lambda^{\alpha,m}_{n,i}(x_i-x_c)^{m1}(y_i-y_c)^{m2}\,, \quad \text{for } 0 \le m_1, m_2 \le L\,. \quad (6)$$

By inserting (5) in eq. (4) we can easily recognize that for the weak–interaction matrix we can write

$$\mathbb{Z}^w\, \boldsymbol{I} = \sum_{m=0}^{M} \sum_{\alpha=x,y,z,\phi} \mathbb{H}^T_{\alpha}\, \Lambda^{R}_{\alpha,m}\, \mathbb{A}_m\, \Lambda^{Q}_{\alpha,m}\, \mathbb{H}_{\alpha}\, \boldsymbol{I}\,, \quad (7)$$

where $\mathbb{A}_m = \{A_m(x_i - x_j, y_i - y_j)\}$, $\mathbb{H}_{\alpha}$ joins the basis (weighting) functions to the unknowns vector $\boldsymbol{I} = \{I_n\}$ and $\Lambda^{R,Q}_{\alpha,m} = \left\{\Lambda^{\alpha,m}_{n,i}\right\}$, where $P_m(z) = Q_m(z)$ or $P_m(z) = R_m(z)$ are considered.

It is worth noting that matrix $\mathbb{A}_m$ is a block Toeplitz matrix, while matrices $\mathbb{H}_{\alpha}$ and $\Lambda^{R,Q}_{\alpha,m}$ are extremely sparse (few valued elements per row).

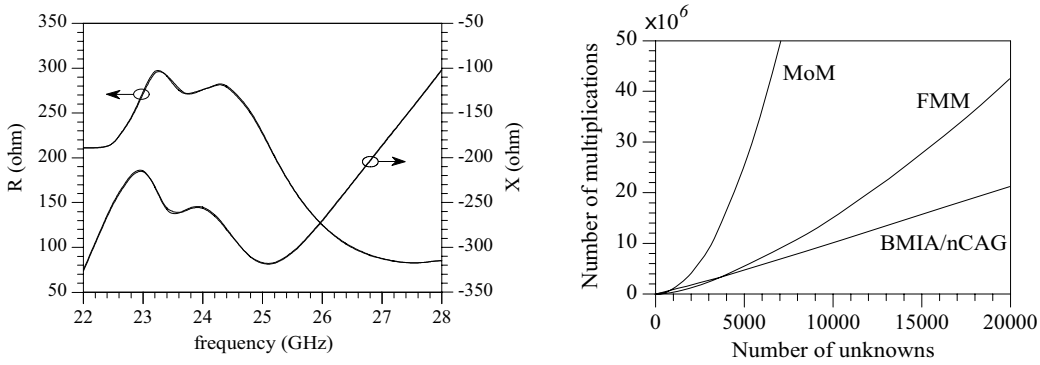
While the $\mathbb{Z}^w$ matrix is a full matrix with $\boldsymbol{O}(N^2)$ elements and usually can not be stored for large–scale problems, we have now to store only $\boldsymbol{O}(4N_g)$ elements for each matrix $\mathbb{A}_m$, and some others matrices that need the overall storage of $\boldsymbol{O}(4(L+1)N_g)$ elements, where N_g is the number of nodes on the regular two–dimensional grid. Furthermore, when the conjugate gradient (CG) method is used to solve the matrix equation $\mathbb{Z}\,\boldsymbol{I} = \boldsymbol{V}$, the $\mathbb{Z}^w\boldsymbol{I}$ product can be conveniently evaluated by performing, for each term m, first the product $\Lambda^{Q}_{\alpha,m}\mathbb{H}_{\alpha}\,\boldsymbol{I}$ (pre–multiplication), then the product $\mathbb{A}_m \cdot (\Lambda^{Q}_{\alpha,m}\mathbb{H}_{\alpha}\,\boldsymbol{I})$ by means of two 2D–FFTs, since $\mathbb{A}_m$ is a block Toeplitz matrix. Finally, the product $\mathbb{H}^T_i\mathbb{R}^{(m)} \cdot (\mathbb{A}_m\, \Lambda^{Q}_{\alpha,m}\mathbb{H}_{\alpha}\,\boldsymbol{I})$ can be performed (post–multiplication). The latter scheme allows to evaluate the $\mathbb{Z}^w\boldsymbol{I}$ product by using $\boldsymbol{O}(N_g\left[8(2\widetilde{M}+1)\log(4N_g) + 6(\widetilde{M}+1)^2\right])$ operations instead of $\boldsymbol{O}(N^2)$.

When we consider the presence in the structure of a ground plane some simplifications arise. As a matter of fact, by applying the image principle we locate image sources

symmetrically with respect to the ground plane, assumed on $z = 0$. This can be taken into account direclty by the H_α matrices. However, since in eq. (4) the term $Q_m(z')$ is proportional to $(z')^{m-1-(\lfloor\sqrt{m-1}\rfloor)^2}$, it is easy to see that when an even power of the source position function z' is considered the presence of an image source leads to annihilate the contribution to the field given by the x and y components of the current density, and by its divergence. In a similar way, contribution given by the z component is annihilated in terms $Q_m(z')$ having an odd power of z'.

NUMERICAL RESULTS

The convergence of the method has been tested by comparing the results obtained with those given by a standard MoM. First, we have analysed the array sketched in Fig. 1 made up of 6×6 stacked patches. Fig. 2a shows the input impedance at the central element of the array evaluated with both the standard MoM procedure and the proposed one. The curves are almost indistinguishable. Fig. 2b shows that the proposed method is profitable, when compared with the FMM, for the analysis of structures that require more than 4000 unknowns.



(a) (b)

Fig. 2 – (a) Real and imaginary part of the input impedance of the central element of the array sketched in Fig. 1 (6777 unknowns, $h_{\max}/r_d = 0.082$); (b) Number of multiplications required for each $(\mathbb{Z}^s + \mathbb{Z}^w)\boldsymbol{I}$ product by MoM, FFM and BMIA/AIM method versus the number of unknowns ($\widetilde{M} = 2$).

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