FAR FIELD RECONSTRUCTION BY A NONREDUNDANT BI-POLAR SCANNING

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Abstract

A sampling interpolation algorithm for reconstructing the field, radiated over a plane in the antenna near-field region from data acquired via a bi-polar scanning, is developed in this work. It is based on the theoretical results relevant to the nonredundant field representations over curves and surfaces. The use of an oblate spheroid as modelling of the source allows to reduce the number of required samples with respect to a spherical modelling when considering quasi-planar antennas. Numerical examples assessing the efficiency and the stability of the algorithm are presented. Such a reconstruction process allows to get an efficient near-field–far-field transformation with bi-polar scanning, which requires a number of data remarkably lower than that needed by the standard approach.

1. INTRODUCTION

As well-known, the evaluation of antenna far-field (FF) from near-field (NF) measurements allows one to overcome those drawbacks which, for electrically large antennas, make unpractical to measure antenna patterns on a conventional FF range. Among the NF-FF transformation techniques, that employing the bi-polar scanning [1, 2] is particularly interesting. In such a scanning the antenna under test (AUT) rotates axially, whereas the probe is attached to the end of an arm which rotates around an axis parallel to the AUT one. This allows to collect the NF data on a grid consisting of concentric rings and radial arcs (see Fig. 1). The bi-polar scanning maintains all the advantages of the plane-polar one [3, 4] while providing a simple and cost-effective measurement system. As a matter of fact, since the arm is fixed at only one point and the probe is attached at its end, the bending is constant and this allows to maintain the planarity. Moreover, rotational movements are preferable to the linear ones, since rotating tables are more accurate than linear positioners. Unfortunately, the original approach in [1, 2] does not take advantage of the nonredundant representations of electromagnetic (EM) fields [5] and, as a consequence, it requires a useless large amount of NF data. Aim of this paper is to develop an efficient NF–FF transformation technique with bi-polar scanning from a nonredundant number of data.

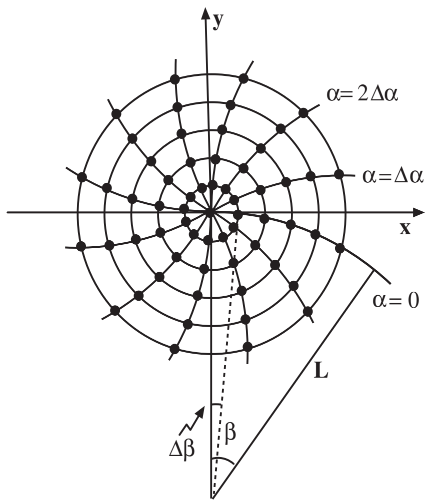

Fig. 1 - Bi-polar scanning.

2. SUMMARY OF PREVIOUS RESULTS

In this section, the theoretical results [5], concerning the nonredundant representation of the EM fields radiated by sources enclosed in arbitrary convex domains with rotational symmetry and observed on surfaces having the same symmetry, are briefly summarized with particular reference to the case of a quasi-planar antenna and a planar observation domain. An effective modelling for quasi-planar sources is obtained by choosing the convex surface Σ enclosing the AUT coincident with an oblate ellipsoid having major and minor semi-axes equal to a and b (see Fig. 2). Since a plane in the NF region can be represented by radial lines and rings, in the following we deal with the field representation on an observation curve C described by an analytical parameterization $\underline{r} = \underline{r}(\xi)$. With reference to C, let us introduce the "reduced" field [5]:

$$\underline{F}(\xi) = \underline{E}(\xi)\, e^{j\gamma(\xi)} \tag{1}$$

where $\gamma(\xi)$ is a phase function to be determined. For large antennas, the "bandlimitation" error, occurring when $\underline{F}$ is approximated by a spatially bandlimited function, becomes negligible as the bandwidth exceeds the critical value

$$W_\xi = \max_\xi\left[w(\xi)\right] = \max_\xi\left[\max_{\underline{r}'}\left|\frac{d\gamma(\xi)}{d\xi} - k\frac{\partial R(\xi,\underline{r}')}{\partial \xi}\right|\right] \tag{2}$$

where k is the wavenumber, $\underline{r}'$ is the generic source point and $R(\xi, r') = |\underline{r}(\xi) - \underline{r}'|$. Therefore such an error can be effectively controlled by choosing a bandwidth equal to $\chi' W_\xi$, $\chi' > 1$ being an excess bandwidth factor. To obtain a nonredundant representation, for each ξ, we must minimize the "local" bandwidth $w(\xi)$. For what concerns the optimal parameter ξ, it must be determined [5] by requiring that $w(\xi)$ is constant. By adopting $W_\xi = k\ell'/2\pi$ (ℓ' being the length of the intersection curve C' between the meridian plane passing through the observation point P and Σ), we get [5]:

$$\gamma = ka\left[v\sqrt{\frac{v^2-1}{v^2-\varepsilon^2}} - E\left(\cos^{-1}\sqrt{\frac{1-\varepsilon^2}{v^2-\varepsilon^2}}\,\middle|\,\varepsilon^2\right)\right]; \quad \xi = (\pi/2)\left[E\left(\sin^{-1}u\,\middle|\,\varepsilon^2\right)\big/E\left(\pi/2\,\middle|\,\varepsilon^2\right)\right] \tag{3}$$

where $E(\cdot|\cdot)$ denotes the elliptic integral of second kind [6], $\varepsilon = f/a$ is the eccentricity of the ellipsoid and $u = (r_1 - r_2)/2f$, $v = (r_1 + r_2)/2a$ are the elliptic coordinates, $r_{1,2}$ being the distances from P to the foci and 2f the focal distance. Moreover,

$$\sin^{-1} u = \vartheta_\infty \tag{4}$$

ϑ_∞ being the polar angle of the asymptote to the hyperbola through P. When the observation curve is a ring, the phase function is constant and it is convenient to use the azimuthal angle φ as parameter. The corresponding bandwidth is [5]:

$$W_\varphi(\xi) = ka\sin\vartheta_\infty(\xi) \tag{5}$$

3. FIELD RECONSTRUCTION OVER A PLANE FROM BI-POLAR SAMPLES

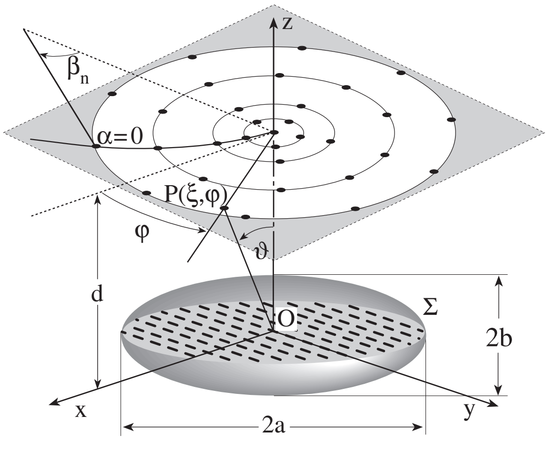

Fig. 2 - Geometry of the problem.

A point on the plane in the NF region can be specified by the bi-polar coordinate system using the AUT angle α, the angle β and the length L of the arm (see Fig. 1). The standard polar coordinates are related to them by the following relations:

$$\rho = 2L\sin(\beta/2)\;; \quad \varphi = \alpha - \beta/2 \tag{6}$$

A convenient way to obtain a non-redundant field representation over a plane from bi-polar samples is to describe it by means of radial lines and rings, as in the plane-polar case. Note that the way of collecting the data in a bi-polar scanning system implies that the starting sample at $\alpha = 0$ on the n-th ring is shifted by $\varphi_0(\xi_n) = -\beta_n/2$ (see Figs. 1 and 2) with respect to the corresponding one in the plane-polar scanning. As a consequence, according to the previous results, the reduced field at any point over the plane can be reconstructed from the bi-polar data via the following optimal sampling interpolation formula:

$$\underline{F}(\xi,\varphi)= \sum_{n=n_0-q+1}^{n_0+q} \left\{ \Omega_N(\xi-\xi_n)\, D_{N''}(\xi-\xi_n) \sum_{m=m_0-p+1}^{m_0+p} \underline{F}(\xi_n,\varphi_{m,n})\,\Omega_{M_n}(\varphi-\varphi_{m,n})\, D_{M''_n}(\varphi-\varphi_{m,n}) \right\} \quad (7)$$

where $n_0 = \mathrm{Int}(\xi/\Delta\xi)$, $m_0 = \mathrm{Int}((\varphi-\varphi_0(\xi_n))/\Delta\varphi_n)$ are the indexes of the sample nearest on the left to the output point, 2q, 2p are the number of retained samples along ξ and φ, and

$$\xi_n = n\Delta\xi = 2n\pi/(2N''+1); \qquad \varphi_{m,n} = \varphi_0(\xi_n) + m\Delta\varphi_n = \varphi_0(\xi_n) + 2m\pi/(2M''_n+1) \quad (8)$$

$$N'' = \mathrm{Int}(\chi N') + 1\ ; \qquad N' = \mathrm{Int}(\chi' W_\xi) + 1\ ; \qquad N = N'' - N' \quad (9)$$

$$M''_n = \mathrm{Int}(\chi M'_n) + 1\ ; \qquad M'_n = \mathrm{Int}(\chi^* W_{\varphi_n}) + 1\ ; \qquad M_n = M''_n - M'_n \quad (10)$$

$$\chi^* = \chi^*(\xi) = 1 + (\chi' - 1)[\sin\vartheta_\infty(\xi)]^{-2/3} \quad (11)$$

$\chi > 1$ being the oversampling factor, needed to control the truncation error. Moreover,

$$D_{M''}(x) = \frac{\sin[(2M''+1)x/2]}{(2M''+1)\sin(x/2)}; \qquad \Omega_M(x) = \frac{T_M\left[2(\cos(x/2)/\cos(x_0/2))^2 - 1\right]}{T_M\left[2/\cos^2(x_0/2) - 1\right]} \quad (12)$$

are the Dirichlet and Tschebyscheff Sampling (TS) functions [4, 5], respectively, wherein $T_M(\cdot)$ is the Tschebyscheff polynomial of degree M. In (12) x_0 is equal to $q\,\Delta\xi$ or $p\,\Delta\varphi_n$,

4. NUMERICAL RESULTS

In the following we show some numerical tests relevant to a uniform planar circular array (see Fig. 1) having diameter $2a = 31.2\ \lambda$, λ being the wavelength. Its elements are elementary Huygens sources linearly polarized along the y axis and are radially and azimuthally spaced of 0.6λ. The array has been modelled as enclosed in an oblate ellipsoid having a = 16 λ and b = 2 λ. The measurement plane is 18 λ away from the antenna center and the bi-polar scanning system is characterized by $\beta_{max} \approx 62°$ and an arm length L = 80 λ, so that the NF data lie in a circular zone of radius $\approx 83\lambda$.

Figure 3 shows a representative reconstruction example of the NF y-component on the radial line at $\varphi = 90°$. As can be seen, the exact and recovered curves are practically indistinguishable. Moreover, to assess in a more quantitative way the algorithm performances, the maximum and mean-square reconstruction errors (normalized to the field maximum value over the plane) have been evaluated for $\chi = \chi' = 1.20$. The corresponding values are reported in Fig. 4. The algorithm stability has been tested by adding random errors to the exact samples. These errors simulate a background noise (bounded to Δa dB in amplitude and with arbitrary phase) and an uncertainty on the field samples of $\pm\Delta a_r$ dB in amplitude and $\pm\Delta\Phi$ degrees in phase. As shown in Fig. 5, the algorithm is stable.

The proposed algorithm has been applied to efficiently recover the plane-rectangular data, needed for the NF–FF transformation. The corresponding E-plane pattern, reconstructed from the recovered plane-rectangular data lying in a $100\lambda \times 100\lambda$ square grid, is shown (crosses) in Fig. 6. The pattern reconstructed from the exact plane-rectangular field samples lying in the same grid is also reported as reference (solid line) in the same figure. As can be seen from this comparison, also the FF reconstruction is very accurate, thus assessing the effectiveness of the developed technique. It may be interesting to compare the number of data (7,266) needed by the proposed technique with that (61,259) required by the standard bi-polar scanning technique [1, 2] to cover the same scanning zone.

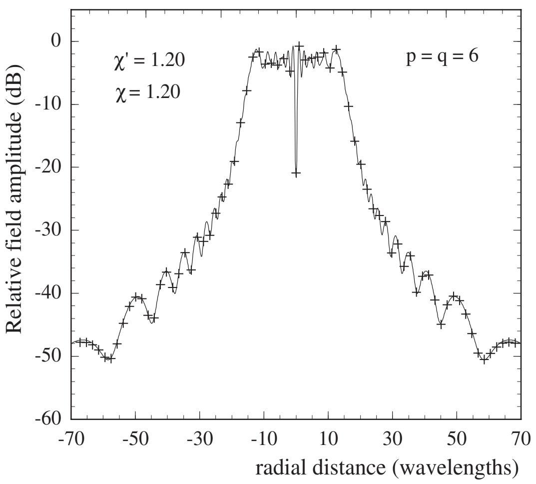

Fig.3 - Amplitude of the NF y-component on the radial line at $\varphi = 90°$. Solid line: exact. Crosses: interpolated.

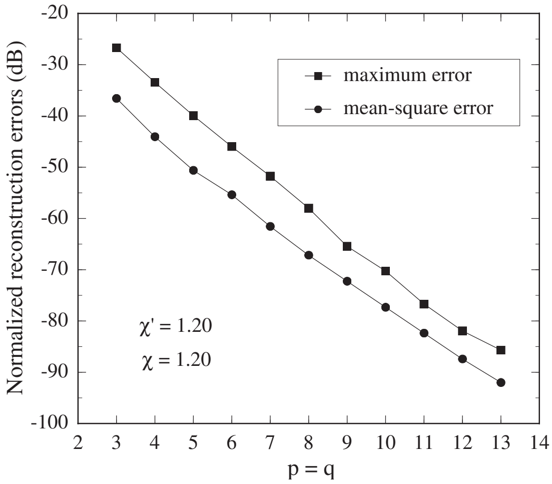

Fig.4 - Normalized reconstruction errors.

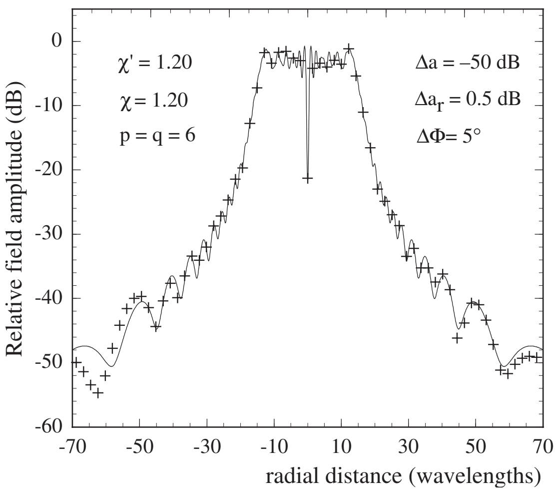

Fig.5 - Amplitude of the NF y-component at $\varphi = 90°$. Solid line: exact. Crosses: interpolated from error affected data.

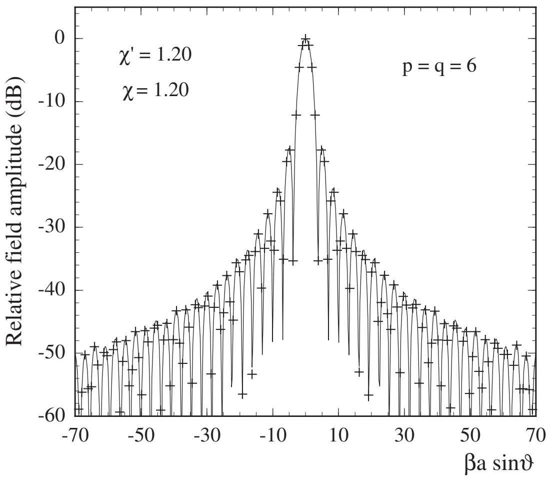

Fig.6 - FF pattern in the E-plane. Solid line: reference. Crosses: reconstructed from bi-polar NF measurements.

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