ELECTROMAGNETIC WAVE SCATTERING FROM A DIELECTRIC SLAB WITH RANDOM MATERIAL PROPERTIES

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Abstract

An investigation of electromagnetic wave propagation in randomly-inhomogeneous media with refractive index fluctuations is performed. The problem is treated both analytically, by solving stochastic differential equations in a statistical sense once the statistical parameters of the dielectric fluctuations are assumed, and numerically, by performing a series of simulation over given, fixed instances of a random medium and by extracting the statistical parameters of the computed field. Numerical simulation are obtained via the Finite Element Method (FEM).

1 - INTRODUCTION

Statistical characteristics of scattered radiation passing through a planar weakly absorptive layer with permittivity fluctuations, when the source and receiver are located in homogeneous media, is considered. Great attention is devoted to the investigation of the influence of the distance between the emitter and the receiver on the broadening of the spatial spectrum and displacement of angular power spectrum maximum of scattered electromagnetic waves. Correlation function of phase fluctuations and spatial spectrum of received radiation have been obtained utilizing ray-optics approximation. It is shown that the width of the spectrum tends to zero when the receiver is located far from the lower boundary of an inhomogeneous plane layer. The spectral maximum is substantially displaced with respect to the direction to the source and changes its sign. The influence of the distances from location of the emitter and antenna with respect to the boundaries of the plane layer of the turbulent plasma, on statistical characteristics of scattered radiation is analysed in [1]. The problem is then treated also numerically, by performing a series of Finite Elements (FEM) analyses over a series of test cases to extract statistical data.

2 - ANALYTIC APPROACH

Point source is assumed to be located in a semi-infinite homogeneous medium characterizing by dielectric permittivity ε_1 at a distance L_1 above the planar inhomogeneous layer. This latter is L_2 thick and has $\varepsilon_{2*}(\mathbf{r}) = \varepsilon_2 + i\varepsilon'' + \varepsilon(\mathbf{r})$, where ε_2 is a homogeneus dielectric permittivity, ε'' characterizes weak absorption ($\varepsilon'' << \varepsilon_2$) and $\varepsilon(\mathbf{r})$ is a random function of coordinate, setting off scattering of electromagnetic waves. Antenna is located at a distance L_3 from the lower boundary of the inhomogeneus layer, in the semi-infinite homogeneous medium below characterized by a dielectric permittivity ε_3. The direction of a wavevector of the incident wave makes an angle θ with the observation line connecting the source and receiver. Expanding field of the pont source (Weyl representation) in plane waves and using the saddle–point method, in a zero approximation (without regarding random inhomogeneities $\varepsilon(\mathbf{r})$=0), the transversal component of one inhomogeneous plane wave giving the major contribution to the wave field at the observation point has been calculated:

$$æ = G\, L_2 \left(\frac{L_1}{L_2} \operatorname{tg}\theta + \frac{\sin\theta}{\sqrt{m_{21}^2 - \sin^2\theta}} + \frac{L_3}{L_2} \frac{\sin\theta}{\sqrt{m_{31}^2 - \sin^2\theta}} \right) \left(1 + i \frac{\varepsilon''\, L_2\, G}{2\, \varepsilon_2\, k_2} \right) \quad (1)$$

where: $G \equiv k_0 \sqrt{\varepsilon_1}\, (L_1 + L_2 \sqrt{m_{12}} + L_3 \sqrt{m_{13}})^{-1}$, $m_{12} = \varepsilon_1 / \varepsilon_2$ and $m_{13} = \varepsilon_1 / \varepsilon_3$.

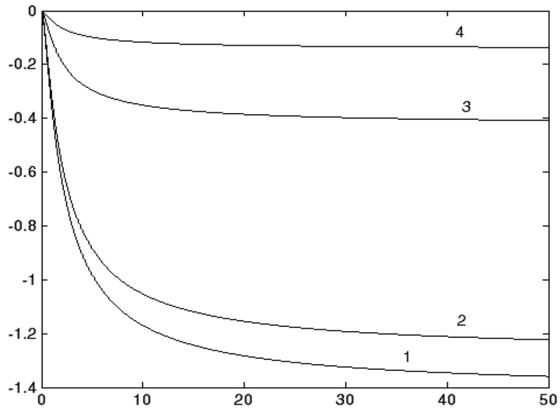

Fig. 1. *Plots of normalized* $\Delta æ_x$ *as a function of* L_1/L_2 *when* $L_3/L_2 = 0$. $1 - \varepsilon'' = 10^{-4}$, $2 - \varepsilon'' = 3\times10^{-4}$, $3 - \varepsilon'' = 9\times10^{-4}$, $4 - \varepsilon'' = 10^{-3}$

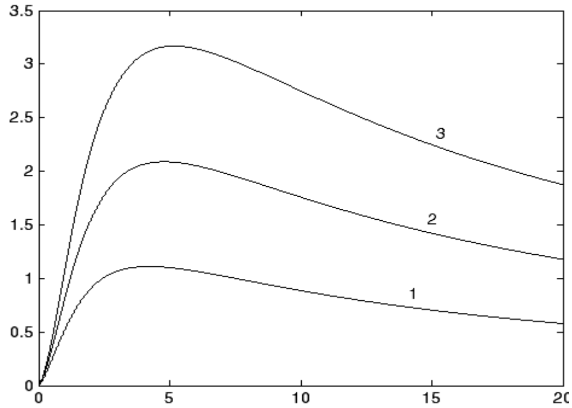

Fig. 2. Plots of normalized $\Delta æ_x$ as a function of L_3/L_2 when $L_1/L_2 = 0$. $1 - \varepsilon'' = 2\times10^{-3}$, $2 - \varepsilon'' = 2.3\times10^{-3}$, $3 - \varepsilon'' = 2.5\times10^{-3}$.

Fluctuations of dielectric permittivity in the layer give rise to fluctuations of wave characteristics at the observation point. The statistical characteristics of the wave field are primarily determined by the complex phase fluctuations (φ_1). The task has been solved using non-stationary ray-optics approximation. Taking dielectric permittivity fluctuations to be sufficiently small, we expand the wave phase characteristics into the series utilizing perturbation method: $\mathbf{k} = \mathbf{k}_0 + \mathbf{k}_1(\mathbf{r}) + \dots$, $\varphi = \varphi_0 + \varphi_1 + \dots$ The fluctuating terms $\mathbf{k}_1$ and φ_1 are proportional to the small dimensionless parameter $\varepsilon / \varepsilon_2 << 1$. The unperturbed wavevector $\mathbf{k}_0$ was obtained for homogeneous medium and its projection on the axis X was determined by the formula (1): $k_x^0 = æ$. Utilizing stochastic eikonal equation, inside the inhomogeneous plane layer the fluctuating term of the complex phase satisfies a linear differential equation

$$æ \frac{\partial \varphi_1}{\partial x} + k_{z2} \frac{\partial \varphi_1}{\partial z} = -\frac{1}{2} k_0^2\, \varepsilon(\mathbf{r}) \tag{2}$$

where: $k_{z2} = \sqrt{k_0^2(\varepsilon_2 + i\varepsilon'') - æ^2}$ is the longitudinal component of the wave vector in the layer. Integrating equation (2) along the complex characteristics, correlation function of complex phase fluctuations when two observation points are spaced apart at a very small distance X has been calculated

$$W_\varphi(X,\ \rho_x,\ 0,\ L_0) = <\varphi_1(X,0,L_0)\ \ \varphi_1^*(X - \rho_x,0,L_0)> \ = \frac{\pi\ k_0^4}{2\,k_{z2}^2} \int_{-\infty}^{\infty} d\,æ_x\ d\,æ_y\ \cdot$$

$$\cdot \int_0^{L_2} d\eta \exp\Big\{ i\, æ_x \Big[\rho_x - \frac{\rho_x}{L_0}(A_1 - A_2) + \eta \frac{\rho_x}{L_0}(C_1 - C_2)\Big] + æ_x\,[B_1 + B_2 -$$

$$-\eta\,(C_1 D_1 + C_2 D_2)\,]\Big\} \times W_\varepsilon\ \Big[æ_x,\ æ_y,\ -\frac{1}{2}æ_x\,(C_1 + C_2)\Big] \tag{3}$$

where $W_\varepsilon(æ_x, æ_y, æ_z)$ is the spatial power spectrum of statistically homogeneous dielectric permittivity fluctuations. A_1, A_2, B_1, B_2, C_1, C_2 and D_1, D_2 are complicated functions of all parameters characterizing the given task.

The knowledge of a field spatial (angular) power spectrum is of the greatest practical importance. It can be obtained by Fourier transformation from the correlation function of the complex field. In the most interesting case of strong fluctuations of the phase $<\varphi_1\varphi_1^*>>>1$, when they are assumed to be normally distributed, the angular power spectrum of received radiation has a Gaussian form. Statistical parameters, characterizing scattered electromagnetic waves such as: $\Delta æ_x = (1/i)(\partial W_\varphi/\partial \rho_x)$ determining the displacement of the spatial power spectrum maximum and $<æ_{1x}^2>=-(\partial^2 W_\varphi/\partial \rho_x^2)$, $<æ_{1y}^2> = -(\partial^2 W_\varphi/\partial \rho_y^2)$ - the widths of the angular spectrum in the XZ and YZ planes, respectively, have been obtained. They can be calculated directly from the expression (3) by differentiation. The derivatives of the correlation function of phase fluctuations W_φ are taken at the point x = y = 0.

Numerical simulations have been carried out using statistically isotropic Gaussian correlation function of dielectric permittivity fluctuations with the following dimensionless parameters:

$$k_0 L_2 = 10^4, \quad \theta = 45^\circ, \quad k_0 \ell = 10 , \tag{4}$$

where ℓ is characteristic spatial scale of dielectric permittivity fluctuations in the layer. The results of numerical calculations are illustrated in graphical form. The widths of all spatial spectra are normalized to the width of the power spectrum of a plane wave $(\ell_1 \to \infty)$, which has passed through the layer without regular absorption $(\varepsilon'' = 0)$. In addition, the maximum displacement is normalized to the maximum displacement of a plane wave that has passed through the layer with the same absorption.

Plots of the maximum displacement as function of L_1 / L_2, when the receiver is located on the lower boundary of the plane layer are shown in figure 1. It can be seen that the effects of displacement of the spectral maximum are obviously revealed at $L_1 / L_2 \geq 5$. At $L_1 / L_2 \geq 30$ statistical characteristic tends to its limiting values.

Plots of the parameters of spatial spectrum versus L_3 / L_2, when the emitter is located on the upper boundary of the layer is illustrated is figure 2. It is ease to see that the displacement of spectral maximum changes its sign in comparison with case $L_3 / L_2 = 0$. This is due to weakly attenuated waves inside the layer arrive at the observation point at greater angles to the normal. Figures also shows that the maximum displacement have pronounced extrema at certain values of ε'' [2, 3].

The analysis of above formulaes allowes us to ascertain the main features of the spatial spectrum for the problem in question. If the emitter and receiver are arranged on the boundaries of the layer of a randomly inhomogeneous medium, with $L_1 = L_3 = 0$ (which is equivalent to their location inside the layer when inverse scattering is neglected), or if they arbitrarily located in which case absorption is absent, the value $\Delta æ_x = 0$ (i.e. the spectral maximum at the observation point) coincides exactly with the direction of the point source (there is no displacement).

The results presented above make it clear that the angular distribution of scattered electromagnetic radiation substantially depends on the location of the transmitter and the receiver relative to the layer. The established patterns may be useful in establishing the principles for remote sensing of the ocean.

3 - NUMERICAL APPROACH

To provide a benchmark for the theoretical approach, and to investigate the phenomenon beyond the limits imposed by the theoretical derivation over the variance of the stochastic fluctuation, a numerical approach combining the FEM method and a Floquet Modal expansion have been exploited.

The theory for the technique has been presented in [4,5] and allows for the computation of the transmission and reflection coefficient for a plane wave impinging over a finite thickness inhomogeneous, lossy dielectric slab. The FEM code, exploiting Floquet modes, is limited to periodic structures, but this is not an issue since the statistical parameter, if computed over simulation performed on cases with varying periodicity, will tend to cancel this effect.

The procedure is then to perform a series of independent FEM analyses over a dielectric slab of given dimensions whose permittivity is given but random and in the form $\varepsilon_{2*}(\mathbf{r}) = \varepsilon_2 + i\varepsilon'' + \varepsilon(\mathbf{r})$. To do so a procedure to devise an instance of the stochastic value $\varepsilon(\mathbf{r})$ must be implemented. Such a procedure must provide a function, which is at the same time random and predictable. This latter for a twofold reason. First, the statistic governing the random function must be fully known and fully under control. Then results must be reproducible, *i.e.* it must be possible to perform the same simulation twice. This is not possible with standard random numbers generated by computers.

The choice made is that of using Perlin noise [6], this is a random noise whose amplitude, frequency and variation can be controlled with few parameters generated by a completely deterministic procedure. Yet the output of the procedure, which is based on the truncation effect of the finiteness of computer's arithmetic, is a random 1D, 2D or 3D noise. Besides the coordinates of the point, an integer seed controls the output of the procedure. If the seed is equal then two separate runs of the procedure produces the same noise map, hence results are predictable. If the seed is different then results are very different but statistical parameters over the domain match, hence the analyses can be used to obtain a meaningful statistic.

References

[1] G.V. Jandieri, V.G. Gavrilenko, N.G. Pirtskhalaishvili, V.G. Jandieri, N.Kh. Gomidze, "Multiple Scattering of Electromagnetic Waves in Layer of Magnetized Turbulent Plasma," ICEAA 99, Sept. 13-17, 1999 Torino, Italy, pp.509-511.

[2] Jandieri G.V., Gavrilenko V.G., Semerikov A.A.. "On the Effect of Absorption on Multiple Wave Scattering in Magnetized Turbulent Plasma". *Waves in Random Media*, Vol. 9 1999, po. 427-440.

[3] Jandieri G.V., Gavrilenko V.G., Aistov A.V. "Some Peculiarities of Wave Multiple Scattering in Statistically Unisotropic Medium". *Waves in Random Media*, Vol. 10, No. 4, 2000, pp. 435-445.

[4] G. Pelosi, A. Cocchi, S. Selleri, "Electromagnetic scattering from infinite periodic structures with localized Impurities," *IEEE Transactions on Antennas Propagation*, Vol. 49, No. 5, May 2001, pp. 697-702;

[5] G. Pelosi, R. Coccioli, S. Selleri, *Quick Finite Elements for Electromagnetic Waves*, Artech House, London, June 1998. ISBN 0-89006-848-8.

[6] D.S. Ebert, F.K. Musgrave, D. Peachey, K. Perlin, S. Worley, *Texturing and modeling, a procedural approach*, AP Professional, 1998