# Short-Pulse Radiation by Periodic Arrays using Time Domain Floquet Waves and the FDTD method

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**Abstract**

A network-oriented formulation is investigated here, which extends useful network and transmission line constructs from the frequency domain to the time domain (TD). TD-Floquet Wave (FW)-based modal voltage and current sources on TD-FW transmission lines, and their TD voltage/current responses, are constructed from the vector field equations and, with proper normalization of the dispersive TD-FW eigenmodes, are evaluated in closed form for an infinite pulsed array in free space. This convenient representation is combined with the FDTD algorithm to determine the pulsed modal current generators for each mode (Floquet wave). Only a few TD-FWs are necessary to reconstruct the transient field, in analogy to the frequency domain counterpart.

## I. Introduction

Wide-band and short-pulse radiation from actual rectangular phased array antennas, infinite and truncated periodic structures, frequency selective surfaces and related applications is a topic of increasing interest. Prototype studies of short-pulse radiation by dipole-excited infinite [1] and truncated [2] periodic line arrays, and by infinite [3] and semi-infinite [4] periodic planar arrays form the basic building blocks for the efficient modeling of time dependent radiation from, or scattering by, such periodic structures. These problem-matched modelings pertaining to spatial periodicity furnish understanding of the Time Domain-Floquet Wave (TD-FW) critical parameters and the corresponding phenomenologies. The approach is based on the exact equivalence between summation over the contributions from individual sequentially pulsed dipole elements in an array, and their collective treatment via Poisson summation in terms of a doubly infinite series of TD-FWs. These canonical problems have been solved using exact constructions as well as asymptotics, directly in the TD, when possible [1]-[3], or via initial formulation in the frequency domain (FD) with subsequent inversion to the TD. Asymptotic inversion from the FD yields the instantaneous frequency behavior which parameterizes the constituents TD-FWs in terms of TD-FW footprints generated by the instantaneous frequencies and corresponding wavenumbers. On the other hand, in [5], it is shown how the FDTD method can be used efficiently to model wide band excitation of rectangular waveguides. Pursuing their numerical approach, the FDTD algorithm is applied in [6], [7] to the array element unit cell in an infinite array for computation of the pulsed modal current generators that form the initial conditions for TD-FW propagation in the nonphased case. In [6], the transient radiated field for nonphased arrays is expressed as a sum of TD-FWs with amplitudes given by the modal voltages and currents for construction of an efficient field representation for this class of problems. For instance, transient radiation from a wideband array of bowtie antennas can be obtained by summing only a few TD-FWs. The efficiency and the accuracy of the method described in [7] depend on the dimension of the FDTD computational domain, and on how many FW modes must be used in the signal reconstruction within the frequency range of interest. Here, for the first time at the National Italian Conference, we present a brief review of this long-term planned research activity. Furthermore, we analyze the orthogonality and completeness properties pertaining to the TD-FW as basis sets, and we show new results obtained using the combined (TD-FW)-FDTD algorithm applied to radiating array geometries. The proposed algorithm is advantageous also for treating active array antennas, or other nonlinear planar devices like array mixers, etc. Moreover, it is shown how data obtained from the FDTD modeling of an infinite array can be used to approximately predict short-pulse radiation by finite (truncated) periodic arrays.

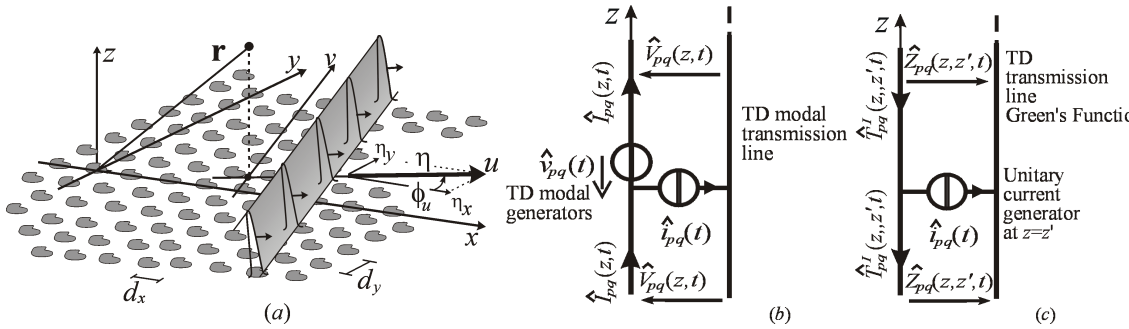


Fig. 1. Generic infinite planar periodic array geometry of elementary radiators, and TD-TL schematizations of the FW-based modal fields and Green's functions. (a) Array geometry. $d_x$, $d_y$: interelement spacing along $x$ and $y$, respectively; $\eta\omega/c = k\eta$: phase gradient of the excitation (i.e., the wavefront) along the direction $\mathbf{1}_u$(see [3]); $v_u^{(p)} = c/\eta$: phase speed along $\mathbf{1}_u$. (b) Equivalent TD transmission line for $pq$th FW, with modal voltage $\hat{V}_{pq}$ and current $\hat{I}_{pq}$, excited by modal voltage and current sources $\hat{v}_{pq}$ and $\hat{i}_{pq}$. (c) TD transmission line voltage and current Green's functions $\hat{Z}(z,z',t)$ and $T^I(z,z',t)$ excited by a unit *current* generator at $z = z\prime$.

## II. Statement of the Problem. Periodicity Conditions in FD and TD

We consider the generic infinite periodic array geometry shown in Fig.1a, with periodicities $d_x$ and $d_y$ along the $x$ and $y$ directions, respectively; the corresponding TD transmission line (TL) representations for the FW-based modal fields and Green's functions are schematized in Figs.1b and 1c, respectively. Concerning notation, a caret ˆ tags time-dependent quantities; bold face symbols define vector quantities; $\mathbf{1}_x$, $\mathbf{1}_y$ and $\mathbf{1}_z$ denote unit vectors along $x$, $y$, and $z$, respectively; the observation point is denoted by $\mathbf{r} = \boldsymbol{\rho} + z\mathbf{1}_z$, with $\boldsymbol{\rho} = x\mathbf{1}_x + y\mathbf{1}_y$. The FW-based modal FD and TD fields due to the array are related by the Fourier transform pair $f(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \hat{f}(\mathbf{r},t)e^{-j\omega t}dt$, $\hat{f}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\mathbf{r},\omega)e^{j\omega t}d\omega$, in which $f$ can be either a scalar or a vector quantity. These Fourier -transform-related fields satisfy the respective FD and TD periodicity conditions

$$\mathbf{E}(\mathbf{r}+\mathbf{d},\omega) = \mathbf{E}(\mathbf{r},\omega)e^{-j\eta(\omega/c)(\mathbf{1}_u\cdot\mathbf{d})}, \qquad \hat{\mathbf{E}}(\mathbf{r}+\mathbf{d},t) = \hat{\mathbf{E}}(\mathbf{r},t-\eta(\mathbf{1}_u\cdot\mathbf{d})/c) \tag{1}$$

where $\mathbf{d} = d_x\mathbf{1}_x + d_y\mathbf{1}_y$, $k = \omega/c$ denotes the ambient wavenumber and $c$ denotes the ambient wavespeed. In the FD, the composite linear phasing on the array is along the direction $\mathbf{1}_u$, perpendicular to $\mathbf{1}_v = \mathbf{1}_z \times \mathbf{1}_u$ (see Fig.1a), with *projected* phasings $\omega\eta_x/c$ and $\omega\eta_y/c$ along the $x$ and $y$ directions, respectively. In the TD, this translates into intercell excitation delayed by $\eta(\mathbf{1}_u\cdot\mathbf{d})/c$. The important nondimensional *single* parameter

$$\eta = \sqrt{\eta_x^2 + \eta_z^2} = c/v_u^{(p)}, \tag{2}$$

which is tied to the rotated coordinate system defined by $u$ (see Fig.1),combines both phasings $\eta_x$ and $\eta_z$. In (2), $v_u^{(p)} = c/\eta$ is the impressed phase speed along $u$. The TD cutoff condition $\eta = 1$ ($v_u^{(p)} = c$) separates two distinct wave dynamics. Here, we treat the case $\eta < 1$ which implies excitation phase speeds $v_u^{(p)} = c/\eta$ (and corresponding *projected* phase speeds $c/\eta_x$ and $c/\eta_z$) larger than the ambient wavespeed $c$.

## III. Time-Domain Modal Representation of the FW-based Fields and Their Sources: TD Modal Transmission Line Fields and Green's Functions

The TD-TL representations are here obtained by Fourier-transforming their FD counterparts. Though in [8] a network formulation for the more complete case of phased arrays ($\eta > 0$) is treated, here we limit the following discussion to the simpler *non*phased case $\eta = 0$, which has also been implemented numerically in Sec.IV. Thus, the transverse (to $z$) TD field is expressed in terms of a complete orthogonal eigenvector set comprising both $E$ (TM) and $H$ (TE) mode functions $\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho})$ and $\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho})$,

$$\hat{\mathbf{E}}_t(\mathbf{r},t) = \sum_{p,q} \hat{V}_{pq}(z,t)\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho}), \qquad \hat{\mathbf{H}}_t(\mathbf{r},t) = \sum_{p,q} \hat{I}_{pq}(z,t)\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho}), \tag{3}$$

where the summation extends over both $E$ and $H$ modes (for the $\eta > 0$ case, the eigenfunctions are time-dependent and the product between network parameters and eigenfuctions has to be replaced by a time convolution, which can be implemented in closed form [8]). For the nonphased case $\eta = 0$, the two TE and TM nondispersive $(p, q) = (0, 0)$ TD-FWs degenerate into a TEM mode. As in the FD case, excitations by TD modal sources $\hat{\mathbf{J}}_{te}(\mathbf{r}, t)$ and $\hat{\mathbf{M}}_{te}(\mathbf{r}, t)$ are likewise represented in terms of this eigenbasis, with amplitudes given by the strengths $\hat{i}_{pq}$ and $\hat{v}_{pq}$ of current and voltage generators: $\hat{\mathbf{J}}_{te}(\mathbf{r}, t) = \sum_{p,q} \hat{i}_{pq}(z, t)\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho})$, $\hat{\mathbf{M}}_{te}(\mathbf{r}, t) = \sum_{p,q} \hat{v}_{pq}(z, t)\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho})$. The eigenfunctions are

$$\hat{\mathbf{e}}^E_{pq}(\boldsymbol{\rho}, t) = \frac{j\boldsymbol{\alpha}_{pq} e^{-j\boldsymbol{\alpha}_{pq}\cdot\boldsymbol{\rho}}}{\sqrt{d_x d_y}}, \qquad \hat{\mathbf{e}}^H_{pq}(\boldsymbol{\rho}) = \hat{\mathbf{e}}^E_{pq}(\boldsymbol{\rho}) \times \mathbf{1}_z, \qquad \boldsymbol{\alpha}_{pq} = \frac{2\pi p}{d_x}\mathbf{1}_x + \frac{2\pi q}{d_y}\mathbf{1}_y \tag{4}$$

where $\boldsymbol{\alpha}_{pq}$ is the transverse-to-$z$ $pq$th FW wavenumber (for the case $\eta > 0$ the transverse wavenumber is dispersive, and for a convenient normalization of the eigenfunctions, see [8]). The magnetic mode functions are given by $\hat{\mathbf{h}}_{pq} = \mathbf{1}_z \times \hat{\mathbf{e}}_{pq}$. For both $E$ and $H$ modes, the TD-TL voltages $\hat{V}_{pq}(z, t)$ in (3) are obtained by superposing contributions from appropriate *point* voltage and current generators distributed along $z'$: $\hat{V}_{pq}(z, t) = -\int dz' \hat{T}^V_{pq}(z, z', t) \otimes \hat{v}_{pq}(z', t) - \int dz' \hat{Z}_{pq}(z, z', t) \otimes \hat{i}_{pq}(z', t)$, where $\otimes$ denotes time convolution. Solutions for the TD-TL Green's functions $\hat{Z}_{pq}(z, z', t)$ and $\hat{T}^V_{pq}(z, z', t)$, excited by delta function current and voltage generators at $z'$ (see Figs.1b,c), are found via Fourier inversion from the FD solutions (see [6], and [8]), which can be evaluated in closed forms in terms of Hankel functions and Incomplete Lipschitz-Hankel Integrals [9]. The TD modal voltage and current generator strengths $\hat{v}_{pq}(t)$ and $\hat{i}_{pq}(t)$ are found as before by projecting the total equivalent electric and magnetic TD currents onto each FW mode: $\hat{v}_{pq}(z, t) =< \hat{\mathbf{M}}_{te}(\mathbf{r}, t); \hat{\mathbf{e}}^\dagger_{pq}(\boldsymbol{\rho}) >$, $\hat{i}_{pq}(z, t) =< \hat{\mathbf{J}}_{te}(\mathbf{r}, t); \hat{\mathbf{e}}^\dagger_{pq}(\boldsymbol{\rho}) >$, where the inner product is defined as $< \mathbf{e}_{pq}; \cdot\mathbf{e}_{p'q'} >= \int_0^{d_x}\int_0^{d_y} \mathbf{e}_{pq} \cdot \mathbf{e}^*_{p'q'} dxdy$ and $^*$ denotes the complex conjugate. For the nonphased case $\eta = 0$, the eigenfunction $\hat{\mathbf{e}}^\dagger_{pq}(\boldsymbol{\rho})$ differs from the one in (3) by a normalization constant (see [8] for the dispersive phased case $\eta > 0$).

## IV. FDTD Modeling of the Single Array Element for the Evaluation of the TD Voltage and Current Generators

In the combined TDFW-FDTD method, the elementary cell is meshed over a finite grid. Periodic Boundary Conditions (PBC) truncate the computational domain at the periodicity faces (Fig.2a) while Absorbing Boundary Conditions are enforced on the remaining upper and lower sides. In the example in Fig.2a, the antenna input port is fed by a wide-band signal, typically a gaussian pulse, a differentiated gaussian pulse or a modulated gaussian pulse. The procedure to efficiently evaluate the transient field at an arbitrary distance from the array plane is described in Fig.2b. The equivalent transient current distribution is calculated at a reference plane $z'$ (see Fig.2a) using the FDTD method. The current generators $\hat{i}_{pq}(t)$ are then evaluated automatically during the FDTD run by numerical projection of equivalent FDTD currents onto the eigenfunctions $\hat{\mathbf{e}}_{pq}$ sampled at the FDTD grid points. Since real TD quantities are required for efficient FDTD processing, *real* valued eigenfunctions are obtained by $(\pm p, \pm q)$-pairing of the eigenfunctions in (4)(see [1], [3],[6]). A convolution of each current generator with the proper TD-TL Green's function gives the field at the required position (see text after (4) and Fig.2b). Generally, only a few TD-FWs are necessary to reconstruct the field at any arbitrary space time location $z, t$ away from the array plane. The number of FW modes which are required for an accurate synthesis of the radiated field are those whose cut-off frequencies ($f^{cutoff}_{pq}$) are smaller than the maximum frequency of interest in the FDTD analysis ($f_{\max} = c/(10\Delta)$ with $\Delta$ denoting the cell's size). Denoting the highest mode orders by $p_{\max}$ and $q_{\max}$, the FDTD grid size should be chosen such that $\Delta \leq \min\{c/(10 f_{\max}), d_x/(20 p_{\max}), d_y/(20 q_{\max})\}$ [7].

As an example, in Fig.2c, the method has been applied to the determination of the transient field radiated by a periodic array (square lattice with $d_x = d_y = 4cm$) of planar self-complementary bow-tie antennas with $\pi/2$ flare angle (see also [6]). The array is expected to radiate a TEM mode over a wide band. The transient voltage generator (see second block in Fig.2c) is a gaussian pulse $\hat{v}_g(t)$ with 20% bandwidth equal to 20GHz (i.e. $v_g(f = 20\text{GHz}) = 0.2 v_g(f = 0\text{Hz})$), and the geometry has been meshed on a cubic grid with side $\Delta = 1mm$. The TD-FW current generators $\hat{i}_{pq}(t)$ have been evaluated at $z' = 5\Delta$, corresponding to 5 FDTD cells away from the array plane. As

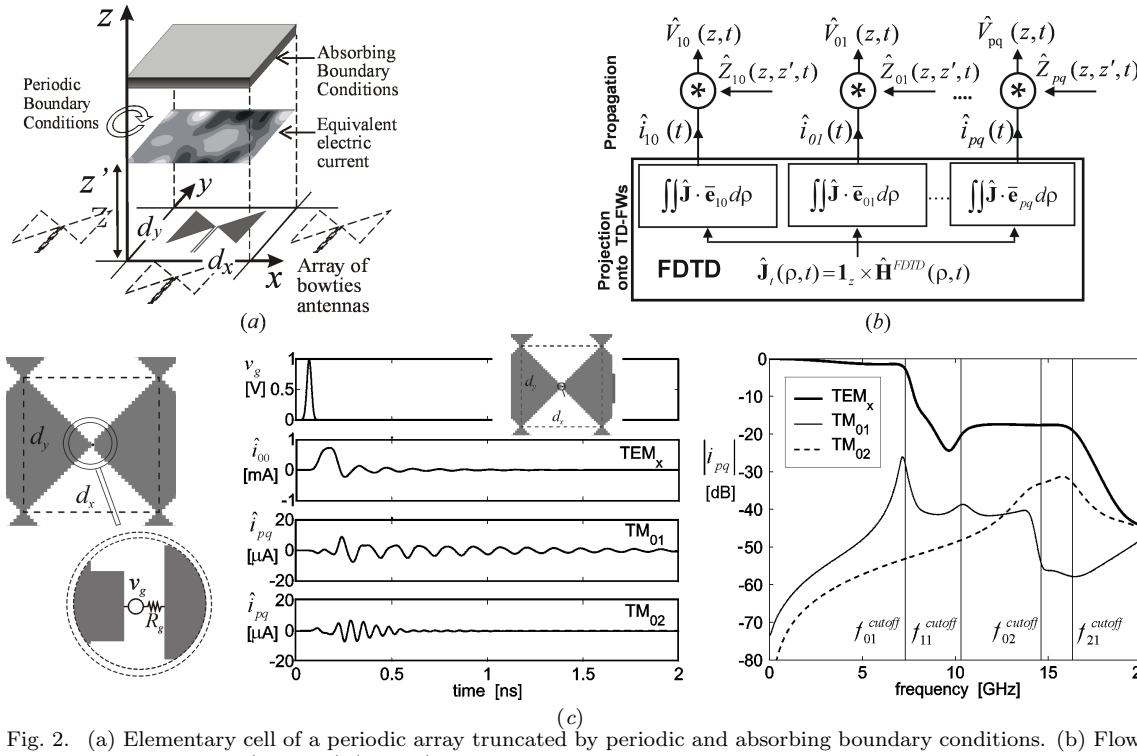


Fig. 2. (a) Elementary cell of a periodic array truncated by periodic and absorbing boundary conditions. (b) Flow-chart for the combined (TD-FW)-(FDTD) algorithm for efficient evaluation of the transient field at an arbitrary distance $z$ from the array plane. Equivalent current distributions are evaluated at a reference plane $z'$ using the FDTD method; equivalent current generators $\hat{i}_{pq}(t)$ are evaluated at $z'$ by projecting the FDTD results onto a modal basis (4); fields are evaluated at $z$ using (3). Only a few TD-FWs are necessary to reconstruct the field at arbitrary $z$ and $t$.(c) Array of Bowtie antennas: Geometry of the FDTD discretization and excitation; pulsed voltage generator $\hat{v}_g(t)$ and TD-FW equivalent current generators $\hat{i}_{pq}(t)$ evaluated at $z = 5mm$ (5 FDTD cells) away from the array plane at $z = 0$; Normalized spectra of the equivalent current generators.

can be observed, the computed $\mathrm{TEM}_x$ current generator reproduces a broader replica of the gaussian pulse and then decays as a *sinc*-like oscillation. The generators $\hat{\imath}^E_{01}(t)$ and $\hat{\imath}^E_{02}(t)$, after an initial transient, oscillate at the cutoff frequencies $f^{cutoff}_{01}$ and $f^{cutoff}_{02}$ (see [3],[6]), slowly attenuating due to the finite length of the excitation waveform $\hat{v}_g(t)$. The spectra of the equivalent current generators normalized by the spectrum of the exciting voltage are shown in the third block of Fig.2c. The TEM spectrum is rather flat from DC to the the first cutoff frequency $f^{cutoff}_{01}$.

The field radiated by a *truncated* array can be evaluated approximately using a Kirchhoff approximation. This consists of truncating the TD-FWs and adding transient FW-modulated diffracted fields arising from the array truncations, as shown in [4].

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