NEAR FIELD–FAR FIELD TRANSFORMATION FROM IRREGULARLY SPACED PLANE-POLAR SAMPLES

F. Ferrara [1], C. Gennarelli [1], G. Riccio [1], C. Savarese [2]

[1] D.I.I.E. - Università di Salerno, via Ponte Don Melillo, 84084 Fisciano (SA)
[2] I.T.T.O.E.M. - Università "Parthenope", via Acton 38, 80133 Napoli

Abstract

An efficient technique to reconstruct the field radiated by an antenna in the far-field region from the knowledge of its nonuniformly spaced plane-polar samples is developed in this work. The Singular Value Decomposition method is applied to evaluate the uniformly distributed samples, whose positions are fixed by a nonredundant sampling representation of the field. Then the near-field data required by the classical plane-rectangular near-field–far-field transformation technique are efficiently evaluated via the Optimal Sampling Interpolation algorithm. As demonstrated by numerical tests, the reconstruction process is accurate and stable with respect to errors affecting the nonuniform samples.

1. INTRODUCTION

An efficient near-field–far-field (NF–FF) transformation technique with plane-polar scanning [1] has been recently developed by considering the antenna under test (AUT) as enclosed in an oblate ellipsoid, which is a source modelling particularly suitable to deal with quasi-planar antennas. Such a technique is based on the theoretical results concerning the nonredundant sampling representations of the radiated electromagnetic (EM) fields [2] and allows to lower in a significant way the number of needed NF data, without losing the efficiency of previous approaches. Unfortunately, due to an inaccurate control of the positioning systems, it may be unpractical to get uniformly spaced NF measurements. On the other hand, the samples position can be accurately read by optical devices. Accordingly, the development of an efficient algorithm, which allows an accurate and stable field reconstruction from the knowledge of the irregularly spaced data (see Fig. 1), becomes relevant. Formulas present in literature for the direct reconstruction from nonuniformly spaced samples are valid only for particular sampling points arrangements, are cumbersome, not user friendly and unstable. Two-dimensional algorithms for recovering the uniform samples from those irregularly spaced on planar, cylindrical or far-field spherical surfaces have been proposed in [3,4]. These algorithms use an iterative technique which converges only if it is possible to build a biunique correspondence between the nonuniform samples and a lattice of regularly spaced ones, by associating at each uniform sampling point the nearest nonuniform one. With reference to a one-dimensional circular domain, this limitation has been overcome in [5] by developing an approach based on the use of the Singular Value Decomposition (SVD) technique [6]. Moreover, it allows one to take advantage of data redundancy for increasing the algorithm stability with respect to unavoidable errors affecting the samples [5].

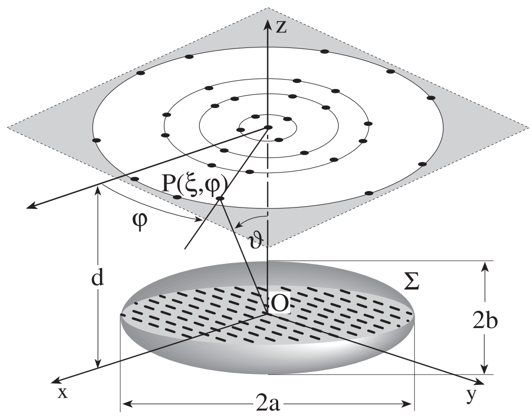

Fig. 1 - Geometry of the problem.

2. SUMMARY OF PREVIOUS RESULTS

In this section we report the key results [1] concerning a nonredundant sampling representation of the EM field radiated by an antenna enclosed in an oblate ellipsoid Σ having major and minor semi-axes equal to a and b (see Fig.1). The field is observed on a NF plane described by radial lines and rings. For each of these curves, denoted by an analytical parameterization $\underline{r} = \underline{r}(\xi)$, the "reduced electric field" $\underline{F}(\xi) = \underline{E}(\xi)\exp(j\gamma(\xi))$, $\gamma(\xi)$ being a phase function to be determined, can be closely approximated by a spatially bandlimited function. For electrically large antennas, the bandlimitation error becomes negligible as the bandwidth exceeds a critical value W_ξ [2]. Therefore such an error can be effectively controlled by choosing a bandwidth equal to $\chi' W_\xi$, $\chi' > 1$ being an excess bandwidth factor. When considering a radial line, by adopting $W_\xi = \beta\ell'/2\pi$ (β being the wavenumber and ℓ' the length of the intersection curve C' between the meridian plane passing through the observation point P and Σ), we get [1]:

$$\gamma = \beta a\left[v\sqrt{\frac{v^2-\varepsilon^2-1}{v^2-\varepsilon^2}} - E\left(\cos^{-1}\sqrt{\frac{1-\varepsilon^2}{v^2-\varepsilon^2}}\,\middle|\,\varepsilon^2\right)\right] \;;\quad \xi = (\pi/2)\left[E\left(\sin^{-1}u\middle|\varepsilon^2\right)\Big/E\left(\pi/2\middle|\varepsilon^2\right)\right] \tag{1}$$

where $E(\cdot|\cdot)$ denotes the elliptic integral of second kind, $\varepsilon = f/a$ is the eccentricity of C', f is its focal distance and $u = (r_1 - r_2)/2f$, $v = (r_1 + r_2)/2a$ are the elliptic coordinates, $r_{1,2}$ being the distances from P to the foci of C'. Moreover, $\sin^{-1}u = \vartheta_\infty$, ϑ_∞ being the polar angle of the asymptote to the hyperbola through P. When the observation curve is a ring, it is convenient to use the azimuthal angle φ as parameter [1,2] and the corresponding bandwidth $W_\varphi(\xi) = \beta a \sin\vartheta_\infty(\xi)$.

By taking into account these results, the field at any observation point $P(\xi,\varphi)$ on the radial line fixed by φ can be evaluated via the corresponding Optimal Sampling Interpolation (OSI) expansion [1]:

$$\underline{F}(\xi,\varphi) = \sum_{n=n_0-p+1}^{n_0+p} \underline{F}(\xi_n,\varphi)\,\Omega_N(\xi-\xi_n)\,D_{N''}(\xi-\xi_n) \tag{2}$$

The uniform samples $\underline{F}(\xi_n,\varphi)$ on the radial line passing through P can be given by:

$$\underline{F}(\xi_n,\varphi) = \sum_{m=m_0-q+1}^{m_0+q} \underline{F}(\xi_n,\varphi_{m,n})\,\Omega_{M_n}(\varphi-\varphi_{m,n})\,D_{M''_n}(\varphi-\varphi_{m,n}) \tag{3}$$

where $\underline{F}(\xi_n,\varphi_{m,n})$ are the uniformly spaced samples on the ring fixed by ξ_n. In (2), (3) $n_0 = \mathrm{Int}(\xi/\Delta\xi)$, $m_0 = \mathrm{Int}(\varphi/\Delta\varphi_n)$ and 2p, 2q are the number of retained samples along ξ and φ. Moreover, $D_{L''}(\cdot)$ and $\Omega_L(\cdot)$ are the Dirichlet and Tschebyscheff Sampling (TS) functions, respectively [1,2], where L is equal to $N = N''-N'$ or $M_n = M''_n - M'_n$. At last,

$$\xi_n = n\Delta\xi = 2n\pi/(2N''+1) \qquad N'' = \mathrm{Int}(\chi N') + 1 \;; \qquad N' = \mathrm{Int}(\chi' W_\xi) + 1 \tag{4}$$

$$\varphi_{m,n} = m\Delta\varphi_n = 2m\pi\big/(2M''_n+1) \qquad M''_n = \mathrm{Int}(\chi M'_n) + 1 \;; \qquad M'_n = \mathrm{Int}(\chi^* W_{\varphi_n}) + 1 \tag{5}$$

$$W_{\varphi_n} = W_\varphi(\xi_n) \;; \qquad \chi^* = \chi^*(\xi) = 1 + (\chi'-1)\left[\sin\vartheta(\xi)\right]^{-2/3} \tag{6}$$

3. NF–FF TRANSFORMATION FROM NONUNIFORMLY SPACED SAMPES

Let us suppose that, apart the sample at the center of the scanning plane, the nonuniformly distributed samples lie on rings not regularly spaced (see Fig. 1). This is a realistic hypothesis in a plane-polar facility. Accordingly, the field at the observation point P is obtained as follows. For each ring, SVD is used for evaluating the uniformly spaced samples from the nonuniform ones. When the uniform samples on the rings are so determined, the OSI expansion is employed to determine the intermediate samples on the radial line through P. Since the intermediate samples are nonuniformly distributed on the considered radial line, the field at P is found in analogous way by recovering the regularly spaced intermediate samples again via SVD.

In particular, given a sequence of $K \geq 2M''+1$ nonuniform sampling points on a ring at ξ, the reduced field at each nonuniform sampling point (ξ,η_k) can be determined by applying the OSI expansion (3). The corresponding system can be rewritten in the following matrix form:

$$\underline{\underline{A}}\ \underline{x} = \underline{b} \tag{7}$$

where $\underline{b}$ is the sequence $\underline{F}(\xi,\eta_k)$ of the known nonuniform samples, $\underline{x}$ is the sequence of the unknown uniformly distributed samples $\underline{F}(\xi,\varphi_m)$, and $\underline{\underline{A}}$ is the $K\times(2M''+1)$ matrix, whose elements are given by the weight functions in the considered OSI expansion:

$$a_{km} = \Omega_M(\eta_k - \varphi_m)\, D_{M''}(\eta_k - \varphi_m) \tag{8}$$

Obviously, for a fixed row k, these elements are equal to zero if the index m is out of the range $[m_0(\xi,\eta_k)-q+1, m_0(\xi,\eta_k)+q]$. A solution, which is the best approximation in the least squares sense of the overdetermined linear system (7), is obtained by using the SVD algorithm. When the uniformly spaced samples on the rings are so determined, the OSI expansion (3) is employed to determine the intermediate samples on the radial line passing through P. These intermediate samples result nonuniformly distributed on the considered curve, so that the previous step must be repeated to find the field at P. Obviously, in this last step, the OSI expansion (2) must be considered instead of (3).

In order to minimize the computational effort, since the aim is to reconstruct the data needed for the classical plane-rectangular NF–FF transformation, it is convenient to evaluate the plane-polar samples at the uniformly distributed points (ξ_n,φ_m), where the φ_m values are those corresponding to the outer ring. In such a way, although we reconstruct NF data redundant along φ, the number of SVD on the radial lines is minimized. Once these uniformly distributed plane-polar samples have been determined, the plane-rectangular data can be evaluated by using expansions (2) and (3), this latter properly modified to take into account the redundancy in φ.

4. NUMERICAL RESULTS

Numerical tests assessing the effectiveness of the approach are reported in the following. The simulation refers to a uniform planar circular array having radius equal to $19.8\,\lambda$, λ being the wavelength. Its elements, radially and azimuthally spaced of $0.6\,\lambda$, are elementary Huygens sources linearly polarized along the y axis. Accordingly, an ellipsoidal source modelling with $2a = 40\,\lambda$ and $2b = 5\,\lambda$ has been used. The measurement plane is $22\,\lambda$ far away from the antenna center and the samples lie in a circular zone of radius $\approx 71\,\lambda$.

Figure 2 shows a representative reconstruction example of the NF y-component (the most significant one) on the radial line at $\varphi = 90°$. As it can be seen, there is a very good agreement between the exact curve and the reconstructed one. To assess quantitatively the algorithm performances, the maximum and mean-square reconstruction errors (normalized to the field maximum on the plane) have been evaluated. To save space, only the plot of the maximum error is reported in Fig. 3. As expected, the error decreases up to very low values on increasing the oversampling factor and/or the number of retained samples. The mean-square error curves run about 10 dB lower than the corresponding ones reported in Fig. 3. The algorithm stability has been investigated by adding random errors to the exact samples. These errors simulate a background noise (bounded to Δa dB in amplitude and with arbitrary phase) and an uncertainty on the field samples of $\pm\Delta a_r$ dB in amplitude and $\pm\Delta\Phi$ degrees in phase. As shown in Fig. 4, the reconstruction process is stable. In any case, as previously stated, the stability can be improved by increasing the number of data.

The proposed algorithm has been applied to efficiently recover the plane-rectangular data, needed for the NF–FF transformation. The corresponding E-plane pattern, reconstructed from the recovered plane-rectangular data lying in a $100\lambda\times100\lambda$ square grid, is shown (crosses) in Fig. 5. The pattern reconstructed from the exact plane-rectangular field samples lying in the same grid is also reported as reference (solid line) in the same figure. As can be seen from this comparison, also the FF reconstruction is very accurate, thus assessing the effectiveness of the developed technique.

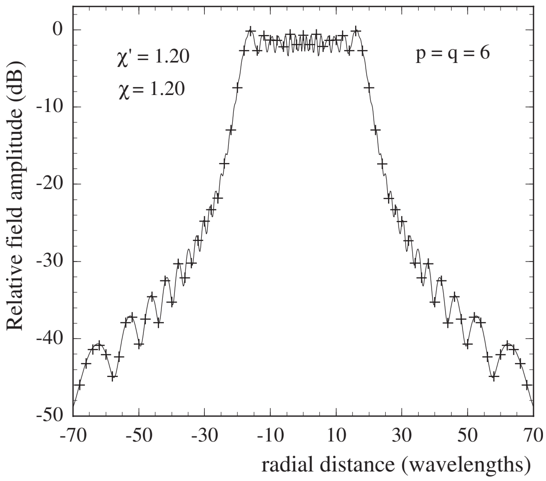

Fig.2 - Amplitude of the NF y-component on the radial line at $\varphi = 90°$. Solid line: exact. Crosses: interpolated.

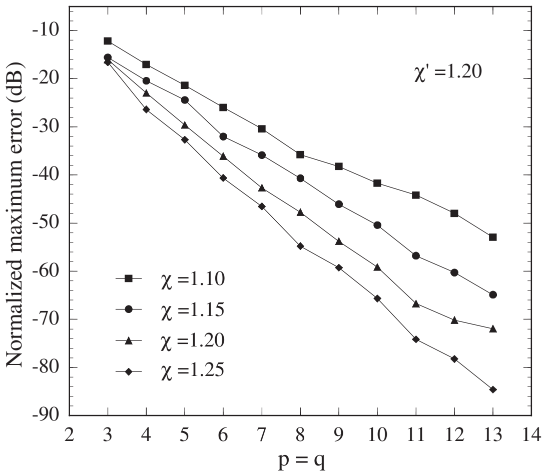

Fig.3 - Normalized maximum error.

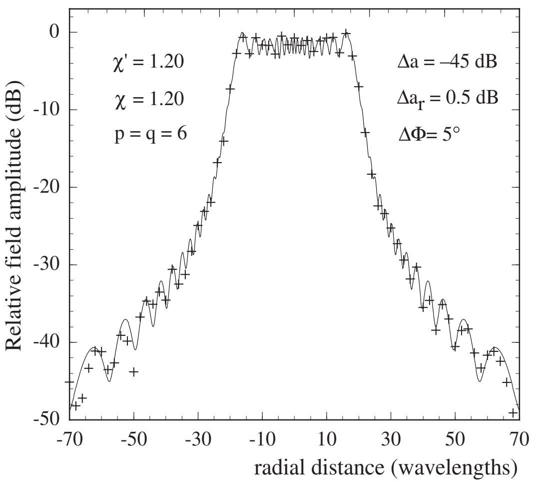

Fig.4 - Amplitude of the NF y-component at $\varphi = 90°$. Solid line: exact. Crosses: interpolated from error affected data.

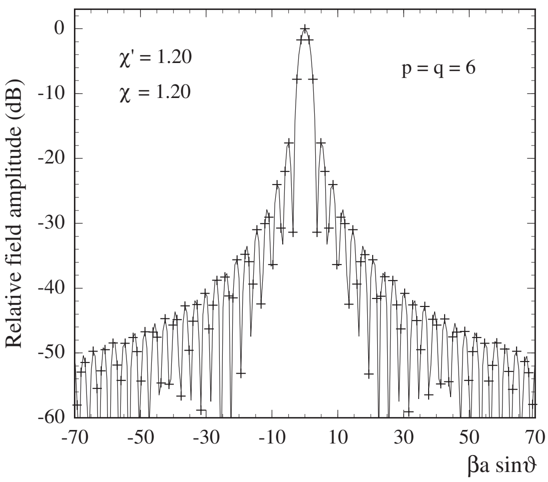

Fig.5 - FF pattern in the E-plane. Solid line: reference. Crosses: reconstructed from nonuniform NF data.

REFERENCES

[1] O.M.Bucci, F.D'Agostino, C.Gennarelli, G.Riccio, and C.Savarese, "NF–FF transformation with plane-polar scanning: ellipsoidal modelling of the antenna," *Automatika*, vol. 41, pp. 159-164, 2000.

[2] O.M.Bucci, C.Gennarelli, and C.Savarese, "Representation of electromagnetic fields over arbitrary surfaces by a finite and non redundant number of samples," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 351-359, 1998.

[3] O.M. Bucci, C.Gennarelli, and C.Savarese, "Interpolation of electromagnetic radiated fields over a plane from nonuniform samples," *IEEE Trans. Antennas Propagat.,* vol. AP-41, pp. 1501-1508, 1993.

[4] O.M.Bucci, C.Gennarelli, G.Riccio, and C.Savarese, "Electromagnetic fields interpolation from nonuniform samples over spherical and cylindrical surfaces," *IEE Proc. – Microwaves Antennas Propagat.,* vol. 141, pp. 77-84, 1994.

[5] F.Ferrara, C.Gennarelli, G.Riccio, and C.Savarese, "Electromagnetic field reconstruction from nonuniformly spaced samples: a SVD based approach," *Proc. of ISMOT 2001*, Montreal, Canada, pp. 483-486.

[6] G.H.Golub, and C.F.Van Loan, *Matrix computations*. J.Hopkins University Press, 1996.