ELEMENTARY DIFFRACTED FIELD CONTRIBUTIONS FOR ARBITRARILY CONTOURED PLANAR ARRAYS

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Abstract

In this paper, a uniform high frequency formulation of the Green's Function for an arbitrarily contoured finite array of electric dipoles is presented. The planar array is thought as a sequence of parallel finite linear arrays and its field is obtained by numeical superposition of the dominant field contributions from each constituting linear array. The linear array is represented as the difference between two spatially shifted semi-infinite linear arrays. The radiation from each semi-infinite linear array is uniformly asymptotically evaluated, to yield a field representation in terms of truncated Floquet waves and their corresponding tip diffracted contributions. This procedure leads to desccribe the total radiated field as the sum of elementary field contributions arising from the actual rim of the planar array, plus elementary truncated Floquet waves.

INTRODUCTION

Recently, a high-frequency formulation of the Green's Function for semi-infinite [1] or sectoral [2] arrays of dipoles has been introduced. It has been found that an efficient representation of the Green's Function (GF) for finite arrays may be obtained when the Floquet Wave (FW) expansion of the infinite array GF is augmented by additional asymptotic ray contributions arising from edges and vertices of the finite structure. This asymptotic formulation can be cast in the form of a generalized GTD ray theory which includes FWs truncated at appropriate shadow boundaries, with corresponding FW-induced edge and vertex diffracted waves.

However, the description in terms of FW-induced edge and vertex diffracted waves becomes cumbersome for arrays with irregular contour, for the intrinsic ambiguity in defining edges and tips. An alternative, more general, representation is proposed in this paper; this alternative approach consists of expressing the total radiated field in terms of contributions from equivalent point sources distributed along the contour of the array.

The planar distribution of sources is regarded as the superposition of suitably-phased uniform-amplitude parallel finite linear arrays; the radiation of each constituting finite linear array is expressed as the sum of truncated FWs relevant to the infinite linear array plus the corresponding diffracted fields at the two end-points. This is obtained by constructing the field radiated by the finite linear array as the difference between the radiation of two semi-infinite linear arrays. In order to provide practical tools for numerical calculations, the spectral integral form is then asymptotically evaluated to yield the relevant truncated FWs and two FW-induced diffracted fields. Next, the global array field is reconstructed by superimposing the dominant asymptotically field contributions arising from the constituting finite linear arrays.

The proposed approach increases the computational efficiency and provides final expressions for the diffracted FWs that are uniformly valid at any observation point. The line-by-line strategy followed here easily allows the introduction of tapering along the direction transverse to the lines, as well as along the lines, by the inclusion of slope diffraction effects.

The accuracy and efficiency of the asymptotic formulation for arbitrarily contoured arrays is discussed and its feasibility and effectiveness is demonstrated by numerical simulations over an eight-sided array.

FORMULATION

The geometry of the problem, which is depicted in Fig. 1(a), shows an arbitrarily contoured array of elementary dipoles directed along the unit vector $\hat{\mathbf{p}}$. They are fed with x-dependent amplitude and

linearly phased currents. The interelement period along x and y are d_x and d_y, respectively, and $k\cos\gamma_x$ and $k\cos\gamma_y$ denote the corresponding linear phase gradients. Two perpendicular periodicity directions have been assumed and a rectangular reference system is defined with its x and y axes parallel to them. It should be noted, however, that this procedure is applicable to arrays with non orthogonal grid as well.

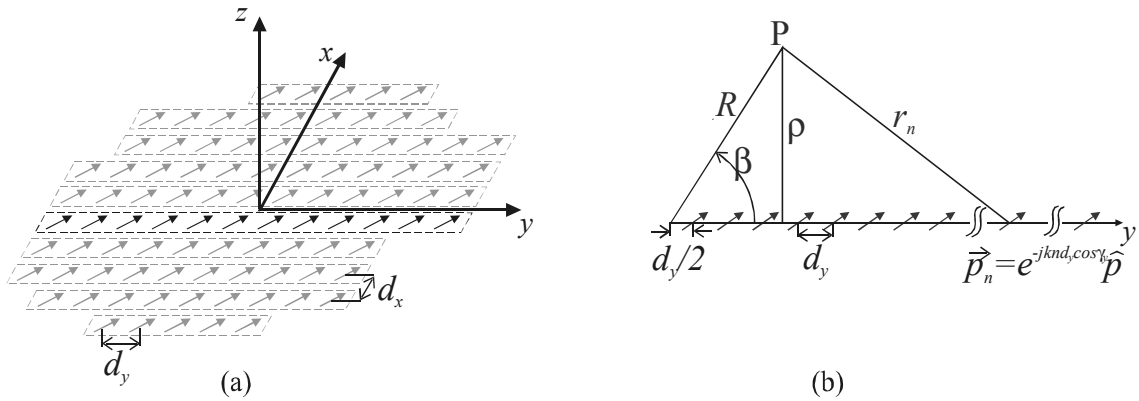

Fig. 1. Geometry of the problem. (a) Arbitrarly contoured planar array and its subdivision into linear subarrays. (b) Semi-infinite linear array.

The electric field radiated by the planar array is obtained as the superposition of the fields radiated by linearly-phased, uniform-amplitude parallel finite linear arrays (Fig. 1). Each one of them is represented as the difference between two spatially shifted semi-infinite linear arrays. The basic constituent of the procedure is therefore the formulation and the relevant asymptotic analysis of the field radiated by a linearly-phased semi-infinite linear array. Cylindrical and spherical local reference systems are introduced with their axes coincident with the axis of the linear array (Fig. 1(b)). For reasons which will be clear afterwards, the origin of the reference systems is located at a point shifted half a period with respect to the first element of the array

For the sake of simplicity in the explanation let us refer to the magnetic vector potential. This can be obtained by direct element-by-element summation over the individual dipole contributions, which can be also rephrased in terms of spectral Fourier counterpart:

$$\vec{A}(P) = \sum_{n=0}^{\infty} \frac{e^{-jkr_n}}{4\pi r_n} e^{-jk\cos\gamma_y n d_y}\hat{\mathbf{p}} = \sum_{n=0}^{\infty} \frac{j}{8\pi} \int_{-\infty}^{+\infty} H_0^{(2)}(k_\rho \rho) e^{-jk_y(y-nd_y-\frac{d_y}{2})} e^{-jk\cos\gamma_y n d_y} dk_y \hat{\mathbf{p}} = A(P)\hat{\mathbf{p}}, \tag{1}$$

where ρ is the distance from the y-axis.

By interchanging the sequence of the n-sum and spectral integration operations, the resulting n-series is expressed in a closed form, obtaining:

$$A(P) = \frac{j}{8\pi} \int_{-\infty}^{+\infty} H_0^{(2)}(k_\rho \rho) \frac{e^{-jk_y y}}{1-e^{jd(k_y - k\cos\gamma_y)}} dk_x = \frac{e^{jk\frac{d_y}{2}\cos\gamma_y}}{16\pi} \int_{-\infty}^{+\infty} H_0^{(2)}(k_\rho \rho) \frac{e^{-jk_y y}}{\sin\left[\frac{d_y}{2}(k_y - k\cos\gamma_y)\right]} dk_y. \tag{2}$$

After using spherical coordinates in the spectral and spatial domains ($k_y = k\cos\theta$, $y = R\cos\beta$) and the approximation for large argument of the Hankel function, eq.(2) becomes:

$$A(P) \cong \frac{k e^{jk\frac{d_y}{2}\cos\gamma_y}}{16\pi} \int_{C_\theta} \sqrt{\frac{2j\sin\theta}{kR\sin\beta}} \frac{e^{-jkR\cos(\theta-\beta)}}{\sin\left[\frac{kd_y}{2}(\cos\theta - \cos\gamma_y)\right]} d\theta. \tag{3}$$

To perform the asymptotic evaluation of eq.(3), the integration contour is deformed into the steepest descent path (SDP) through the pertinent saddle point $\theta = \beta$ (Fig. 3). The high-frequency solution is therefore obtained as the summation of the integral along the SDP and the residues at the poles $\theta_p = \cos^{-1}\left(\cos\gamma_y + 2\pi p / (kd_y)\right)$ intercepted during the path deformation.

The residue contributions represent the cylindrical FW's of the infinite linear array, with their domain of existence terminated by a conical shadow boundary (CSB); the discontinuities at the CSB are uniformly compensated by the tip-induced spherical diffracted wave provided by the SDP integral (Fig. 2 (b)).

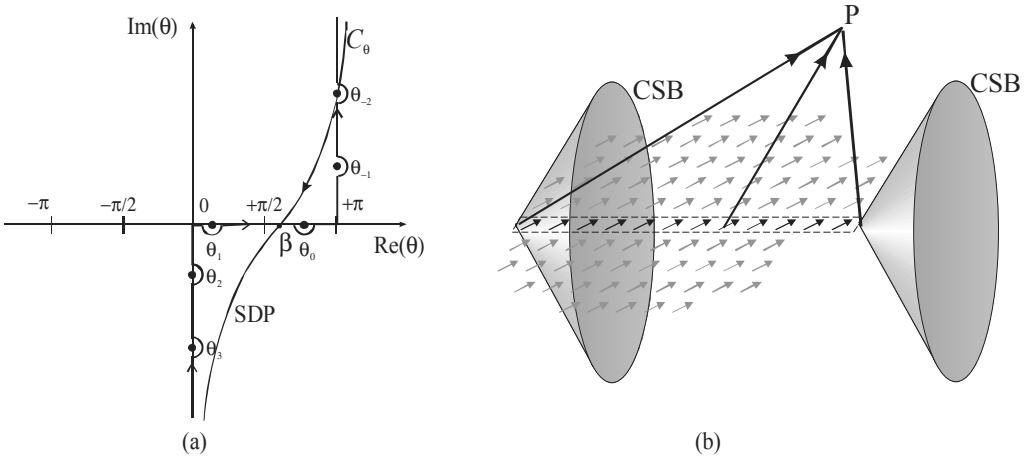

Fig. 2. (a) Contours of integration in the θ-plane. (b) Ray description of the radiation from the array.

The asymptotic evaluation, performed via the Van der Waerden method [3], yields :

$$A(P) \cong 2\pi j \sum_p A_p^{FW} U(\theta_p - \beta) + \frac{1}{16\pi} e^{jk\cos\gamma d_y/2} \frac{e^{-jkR}}{R} \frac{2j}{\sin\left[\frac{kd_y}{2}(\cos\beta - \cos\gamma_y)\right]} +$$

$$+ \frac{1}{16\pi}\sqrt{\frac{2\pi j}{kR}} e^{jk\cos\gamma_y d_y/2} e^{-jkR} \sum_p \frac{(-1)^p}{d_y \sin\theta_p} \left[\frac{G(-\theta_p)}{\sin\left(\frac{\beta+\theta_p}{2}\right)} \left[F\left(\delta_+^2\right)-1\right] - \frac{G(\theta_p)}{\sin\left(\frac{\beta-\theta_p}{2}\right)} \left[F\left(\delta_-^2\right)-1\right] \right] \quad (4)$$

where

$$\delta_\pm = \sqrt{2kR}\sin\left(\frac{\beta \pm \theta_p}{2}\right), \qquad G(\theta) = \sqrt{\frac{2j\sin\theta}{\pi kR \sin\beta}}, \quad (5)$$

A_p^{FW} are the Floquet waves of the infinite linear array:

$$A_p^{FW} = \frac{-jG(\theta_p)}{4d\sin\theta_p} e^{-jkR\cos(\beta-\theta_p)} e^{jkd/2\cos\gamma_y}, \quad (6)$$

$F(\cdot)$ is the UTD transition function [4] and $U(\cdot)$ is the Heavyside unit step function. From (4)-(6), the high-frequency field from the total array is obtained by superimposition of the linear array asymptotic waves.

NUMERICAL RESULTS

The procedure outlined so far has been applied to an eight-sided array of 432 parallel dipoles and compared with a direct element-by-element summation. As a first example, an array with uniform amplitude excitation is now considered. The array is characterized by interelement periods d_x=0.7λ and d_y=0.9λ along x and y, respectively, and by the linear phase gradients $k\cos\gamma_x = k/2$ and $k\cos\gamma_y = k/\sqrt{2}$. Fig. 2 shows the amplitude of the θ- and ϕ- components of the radiated field.

As a second example, a similar array configuration is then considered, with d_x=0.5λ, d_y=0.9λ and a tapered excitation $\vec{J}_{nm} = \cos\left[(n/10.5)^2\right]\cos\left(0.9\sqrt{3}\pi m\right)e^{-j\frac{\pi n}{2}}$, in which the tapering along the direction y

has been obtained by superimposition of two uniform amplitude, linear phase excitations. Fig. 3 shows the relevant amplitude of the θ- and ϕ- components of the radiated field.
In both examples, the agreement between the results from this formulation and the reference solution is quite satisfactory.

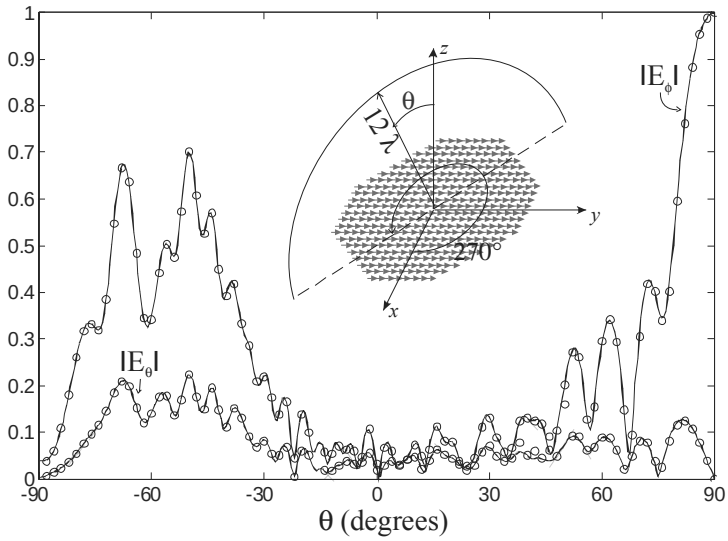

Fig. 3. Normalized electric field for a uniform amplitude octagonal array. The asymptotic solution (circles) is compared with the reference solution (continuous line).

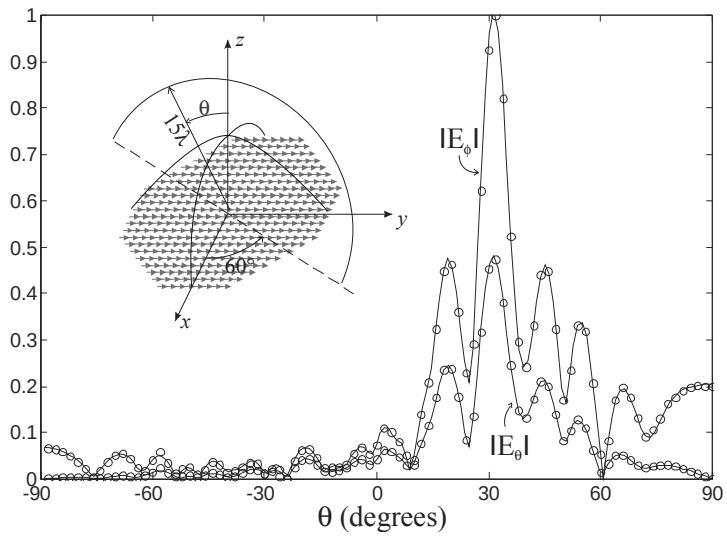

Fig. 4. Normalized electric field for a tapered octagonal array. The asymptotic solution (circles) is compared with the reference solution (continuous line).

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