MULTIRESOLUTION ANALYSIS OF ANTENNAS: TRIANGULAR MESH TESTS

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Abstract

A Multi-Resolution (MR) approach to the EFIE analysis is presented, that keeps the different scales of the problem directly into the basis functions representation. The basis is constructed via the "dual-isoscalar" technique introduced by the authors, and possesses both spectral and spatial resolution, that can be shown to be crucial in handling the condition problem: the obtained MoM matrix is stable against clipping sparsification, and the use of iterative solvers convenient. The presented technique works for both rectangular and triangular meshes: in this contribution test problems meshed with a triangular grid will be considered.

INTRODUCTION

The design-oriented numerical simulation of large and complex printed arrays involves structures that are electrically large yet with fine geometrical details requiring much-smaller than wavelength discretizations. When the EFIE-MoM approach is used, the necessity of representing different scale geometries impacts on the size of the resulting Z-matrix that, together with its poor conditioning are the key limitations of this approach. On the other hand, the presence of different electrical lengths in the structure geometry results in very different scales of variation in the solution. Local interactions in sub-wavelength details, or in proximity of edges and discontinuities, generate small-scale details - with high spatial frequency- in the solution while distant interactions - as well as resonant lengths - are responsible for the low-(spatial)frequency, slow spatial variations.

A number of techniques have been presented in the past years to overcome the above difficulties (see e.g. [1, 2, 4, 5, and ref. therein]).

In [3] authors have been introduced a different approach, based on keeping the different scales directly into the formulation and solution process, a typical Multi-Resolution (MR) instance; this leads to a good control of the conditioning of the matrix, a key in the convergence of iterative solvers, as well as the gateway to sparsification of the matrix. In [3] results of the application of the scheme to rectangular meshes have been presented; here test problems meshed with a triangular grid are considered.

"DUAL-ISOSCALAR" MULTIRESOLUTION SCHEME

The use of "wavelet" functions has generally been limited to the analysis of scalar or 1D problems, due to the intrinsic difficulties of generating and employing "vector" MR (wavelet) functions on 3D or 2.5D structures. The "Dual-IsoScalar" (DIS) approach introduced by the authors in [3] and here only briefly summarized overcomes this limitation.

The generation of the MR basis is approached dividing the unknown surface current into its solenoidal (TE) and quasi-irrotational (quasi-TM, qTM) components, that can be mapped to scalar quantities that posses the same degree of regularity in both (local) spatial directions, and

on which the introduction of wavelet-like constructs is easier. The qTM part is related to the charge density, and thus defined over the cells of the mesh, and the TE part is derived from a "solenoidal potential", defined over the (inner) nodes of the mesh; this duality is present at all stages of the generation of the MR functions.

The MR construction begins by defining a "starting", coarse mesh of the structure, called level $j = -1$, as done with any mesher from the CAD. Subsequent nested domain-decomposition steps are then performed, that are equivalent to the generation of hierarchically nested families of grids of increasing detail; each of these meshes is identified as level-j; the finest such mesh, with $j = L$ is called the "pixel" mesh. On all meshes of level $j \geq 0$, the MR functions are defined via the corresponding scalar quantities: the charge (for qTM) refers to cells of the mesh, the solenoidal potential (for TE) to the nodes. On the scalar quantities, MR is obtained applying "templates" that employ the wavelet constructs; finally, each (TE or qTM) MR scalar function is mapped onto the corresponding vector function. The above is trivial for the TE (it only involves a gradient), and for the qTM it employs the results in [6].

The generation scheme for the functions belonging to levels $j \geq 0$ was described in [3] for a rectangular mesh, but the DIS scheme works for triangular meshes as well, since its scalar "inner nature" does not require separable current components (like cartesian components are in rectangular meshes). Application to triangular grids can be obtained almost directly by application of the described algorithms; in this, we note that the passage from scalar to vector functions relies on the results in [6], that apply indifferently to Rao-Wilton-Glisson (RWG) functions on triangular or rectangular cells.

In [3] it has been shown that for a rectangular grid the $j = -1$ mesh has to be treated differently, since on it, "connecting functions" are defined that link the various sub-domains. In the simplest instance, connection functions are simply the loops (TE) and stars (qTM) of that mesh. However, care must be taken on the type of connecting functions used, since their choice of the connecting functions on the coarsest grid impacts on the overall numerical efficiency, yet not in a dramatic manner (see e.g. [7, 8, 9]). Here the SVD-based "spectral" connecting functions [7, 9] are used.

APPLICATION OF THE MR SCHEME TO TRIANGULAR MESHES

As examples of application of the MR scheme to structures discretized with triangular meshes, we considered here a H-shape patch and a bowtie dipole. In both cases, the solution obtained with the RWG functions defined on the pixel meshes shown in Figg. 1-2 is taken as references.

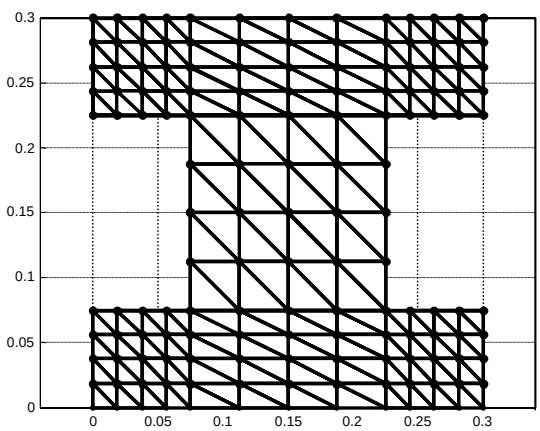

Figure 1. H-shape patch: pixel (finest) mesh

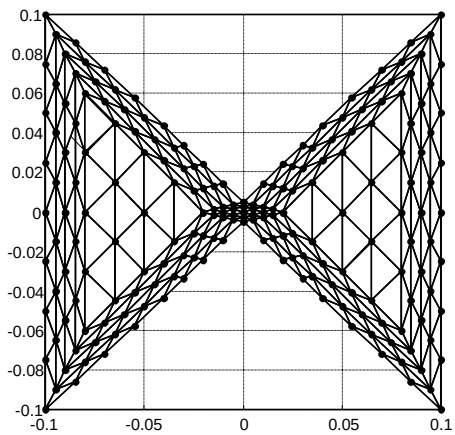

Figure 2. Bowtie dipole: pixel (finest) mesh

On the H-shaped patch there are 304 unknowns; in the MR basis, there are three levels ($L = 1$), and no TE functions at the coarsest level $j = -1$. The MoM matrix in the RWG basis has a condition number of about 700, that drops to about 60 in the MR basis after diagonal preconditioning. This is reflected in the possibility of sparsifying the matrix by clipping entries below a threshold, as seen in Fig. 3; note that while in the RWG basis the error explodes for moderate sparsifications, for the MR basis it remains below 1% up to a sparsity of 75% (25% non-zero entries), which is remarkable for a structure dominated by near-field terms.

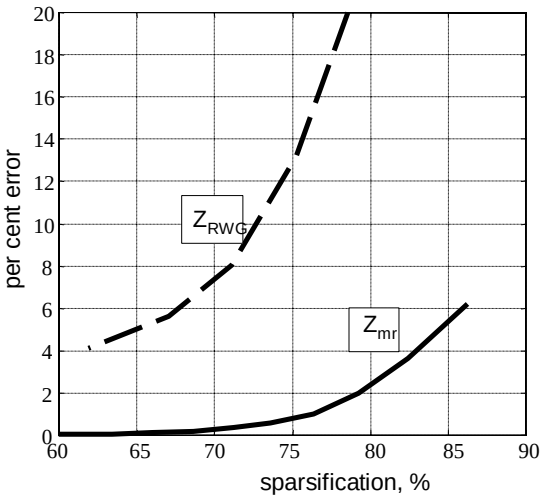

Figure 3. H-shape patch: error on the inverse matrix due to sparsification

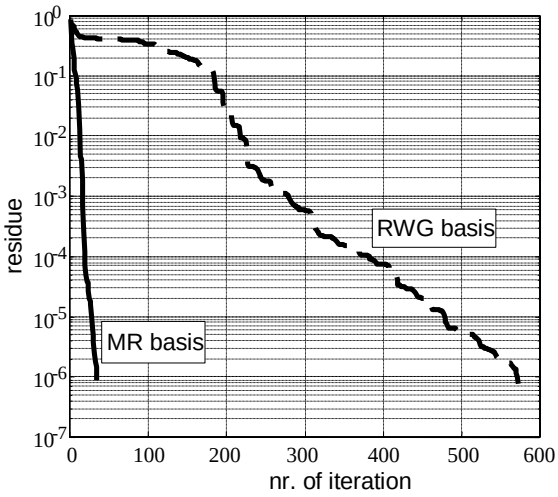

Figure 4. H-shape patch: covergence of Conjugate Gradient

Next, Fig. 4 shows the convergence history of the systems obtained with the RWG and MR bases: the advantage in converegence speed is apparent.

The bowtie dipole has been discretized with 504 basis functions, defined non a non-uniform mesh (see Fig. 2 for the pixel level mesh). Also in this case the MR basis is defined in three levels, but TE functions are present also at level $j = -1$. The bowtie is a broadband antenna (here we have considered the frequency interval [25 MHz $\div$ 1.5 GHz]) and therefore its electrical length, as well as the size of the mesh cells vary strongly from one end to the other of the frequency sweep. On such a structure the use of a MR scheme is particularly meaningful, since at different frequencies functions defined on different level meshes will be predominant.

Because of the large extension of the considered frequency interval, the two-norm condition number κ_{RWG} of the (diagonally preconditioned) MoM matrix in the RWG basis varies from one end to the other of the frequency sweep of almost three order of magnitude ($\kappa_{RWG} \sim 10^6$ at $f_s = 25$ MHz and $\kappa_{RWG} \sim 10^3$ at $f_S = 1.5$ GHz). On the other hand, the use of the MR basis and of the diagonal preconditioner left the condition number κ_{MR} of the resulting matrix almost unchanged on the whole interval. ($\kappa_{MR} \sim 160$ at f_s and $\kappa_{MR} \sim 300$ at f_S). The slight increase of κ_{MR} with the frequency can be ascribed to the increasing of the electrical size of the MR basis functions defined on the coarser mesh, that can become larger with $\lambda/2$: as shown in [9], this fact is responsible of the increase of the condition number.

The frequency behavior of the real and imaginary part of the input impedance of the bowtie are reported in Fig. 5. These plots show that the results obtained in the MR basis, with a matrix sparsification varying from 80% to 90% (corresponding to a clipping threeshold of 10^{-2} at the different frequencies) are almost indistinguishable with the reference solution.

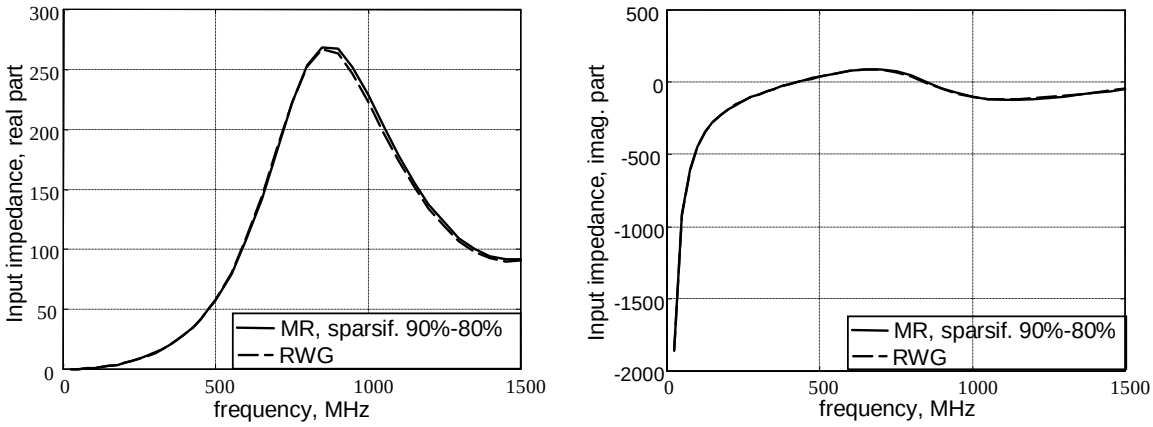

Figure 5. Bowtie dipole: Effect of matrix sparsification on the input impedance. Left: real part. Right imaginary part

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