NEAR FIELD OF A DIPOLE-FED INFINITE SLOT PRINTED BETWEEN TWO HOMOGENEOUS DIELETRICS"

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Abstract

This paper presents a formulation for the Green's function of an electric dipole exciting a slot printed between two different homogeneous media and its relevant asymptotic solution uniformly valid in any space points. Our formulation is based on the representation of a continuous spectrum of modes which has poles at the solution of the dispersion equation, i.e., at the wavenumbers of the leaky guided modes.

INTRODUCTION

In [1] the magnetic current excited by an electric dipole on a slot printed between two different homogeneous media has been derived in analytical form under not restrictive assumption of small slot width in terms of a wavelength. In this paper we continue the investigation on this topic providing a ray description of the field in the denser medium. Although general concerns about Green's function of open waveguiding structures have been addressed by other authors in a more general framework [2][3], the investigation presented here assumes importance becouse of the closed form expressions which simplify the understanding of the physical mechanisms.

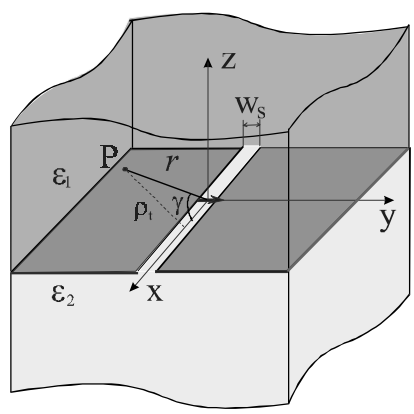


Fig. 1 *Geometry and reference system for an infinite slot excited by an electric dipole. The slot is etched between etched in an infinite ground plane between two different half spaces with dielectric constant ϵ_{r2} (z>0) and ϵ_{r1} (z<0); we assume $\epsilon_{r2}>\epsilon_{r1}$*

MAGNETIC CURRENTS

The geometry, presented in Fig. 1 and consists of an infinite *x*-oriented slot which is printed on an infinite ground plane between two homogeneous dielectric half-spaces of permittivities ϵ_{r2} *(z>0)*, and ϵ_{r1} *(z<0)*, with $\epsilon_{r2}>\epsilon_{r1}$ assumed for convenience. The cross section w_s of the slot is uniform in *x* and small in terms of a wavelength. The structure is excited by a *y*-oriented electric dipole of the same length of the slot width w_s, placed at the origine of the reference system. In addition, ρ_t and *r* are the distances of the observation point from *x* and from the origin, respectively; γ is the angle that the observation direction forms with the positive x axis. The closed form expression for the slot magnetic currents *m(x,y)* derived in [1], is summarized here:

$$m(x,y) = -v(x)\frac{2}{w_s\pi}\left(1-\left(\frac{2y}{w_s}\right)^2\right)^{-1/2} \tag{1}$$

where

$$v(x)= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-jk_x x}}{D(k_x)}dk_x \tag{2}$$

$$D(k_x)=\frac{1}{2k_0\zeta_0}\sum_{i=1}^{2}(k_i^2-k_x^2)J_0\left(\frac{w_s}{4}\sqrt{k_i^2-k_x^2}\right)H_0^{(2)}\left(\frac{w_s}{4}\sqrt{k_i^2-k_x^2}\right) \quad (3)$$

and k_0, ζ_0 are the free-space propagation constant and characteristic impedance, respectively; $k_i = k_0\sqrt{\epsilon_{ri}}$ for $i=1,2$ are the propagation constants in the two media. In (3) J_o is the Bessel function of zero order and $H_0^{(2)}$ is the Hankel function of zero order and second kind.

UNIFORM ASYMPTOTIC FIELD

Since the radiation essentially occurs in the denser medium, the attention will be focused on the potential in medium 2. The x-oriented electric vector potential, associated to the equivalent magnetic currents is

$$F_2 \sim -\int_{-\infty}^{\infty}\frac{e^{-jk_2R(x-x')}}{4\pi R(x-x')}2v(x')\,dx' = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{H_0^{(2)}(\sqrt{k_2^2-k_x^2}\,\rho_t)}{jD(k_x)}e^{-jk_xx}dk_x \quad (4)$$

where $R(x-x')=\sqrt{(x-x')^2+y^2+z^2}=\sqrt{(x-x')^2+\rho_t^2}$. and the factor 2 comes for the application of the image principle (note that for simplicity, we use here a definition of the potential without multiplication for the permittivity). From the potential, it is straightforward to obtain the field .The remaining of this paper will be concerned with the asymptotic approximation of F_2. Indeed, this asymptotic treatment provides the basic physical insight for the comprehension of the vector field mechanisms. Treatment of the vector field can be derived via analytical differentiation.

After asymptotically evaluating the Hankel function in (4) for large values of its argument, the rectilinear spectrum plane k_x, is transformed in the angular spectrum by the change of variable $k_x=k_2\cos\alpha$,

$$F_2 \sim \int_C e^{-jk_2r\cos(\alpha-\gamma)}f(\alpha)\,d\alpha \;;\; f(\alpha)=\frac{-\sqrt{2}j\sqrt{k_2\sin\alpha}}{jD(k_2\cos\alpha)\sqrt{r\pi\,\sin\gamma}\,4\pi} \quad (5)$$

where γ is the angle that the observation vector forms with the positive x axis and C is the contour $(-j\infty, 0, \pi, \pi+j\infty)$, which maps into the angular spectral domain the real axis of the k_x plane. The spectral angular plane is depicted in Fig.1. The phase of the integrand exhibits a saddle point at $\alpha=\gamma$, a branch point at $\gamma_b=\cos^{-1}(k_1/k_2)$ and a pole at $\alpha_{LW}=\cos^{-1}(k_x^{LW}/k_2)$. The branch point due to the square root at k_2 in the rectilinear spectral plane (mapped at points 0 and π in the angular plane) has disappeared owing to the angular transformation used, which automatically solves the ambiguity of $\sqrt{k_2^2-k_x^2}$ defining it as $k_2\sin\alpha$. Note that the log-type singularity at 0 and π still remains; these latter are not however involved in the following asymptotic treatment, as well as the other non-physical poles, which are not depicted in Fig. 1.

To perform the asymptotic evaluation of the integral on the original contour C, we deform this contour onto the steepest descent path (SDP) through the saddle point $\alpha=\gamma$. The SDP equation in the α plane is defined by $Re(\cos(\alpha-\gamma))=1$, with $Im(\cos(\alpha-\gamma))<0$. In this deformation either the leaky wave pole α_{LW} or the log-type branch cut at γ_b may be captured depending on the position of the observer. Three different situations may be defined, which are represented in Fig. 1a,1b,1c, respectively. When $\gamma > \gamma_b$, (Fig 1a) neither α_{LW} nor γ_b are captured during the contour deformation; in this case the total field is reconstructed by the sole SDP integration. For $\gamma < \gamma_b$ (Fig 1b) the branch point γ_b is captured, thus requiring an additional integration around the branch-cut. For $\gamma < \gamma_{sb}$ (Fig 1c) the pole α_{LW} is captured in the contour deformation, thus leading in addition to the SDP and to the branch cut integrations, the pole residue contribution. The boundary between the situation of Figure 1b and 1c occurs when the SDP crosses the pole, that is when $\mathrm{Re}(\cos(\alpha_{LW}-\gamma_{sb}))=1$ which, defining $\alpha_{LW}^{re}=Re(\alpha_{LW})$ and $\alpha_{LW}^{im}=Im(\alpha_{LW})$, implies $\gamma_{sb}=\alpha_{LW}^{re}+\cos^{-1}(1/\cosh(\alpha_{LW}^{im}))$.Eventually, the potential F_2 can be expressed as

$$F_2=(I_{sdp}+I_{LW}U(\gamma_{sb}-\gamma)+I_{lat}\,U(\alpha_b-\gamma)) \quad (6)$$

where $U(x)$ is the unit step function ($U(x)=1$ for $x>0$ $U(x)=0$ for $x<0$) which accounts for the existence domain of the various contributions as described in Fig.s 1a,b,c. In accordance with the non-uniform ray description in [2], the mathematical definition and the corresponding physical meaning of the various contributions is

i) I_{sdp} = SDP integral $\rightarrow$ space wave contribution ($\rightarrow$ F^{space})
ii) I_{lat} = integral around the log-branch cut at α_b $\rightarrow$ lateral wave contribution ($\rightarrow$ F^{lat})
iii) I_{LW} = residue at α_{LW} $\rightarrow$ leaky wave contribution ($\rightarrow$ F^{LW})

In (6), the unit step functions $U(\gamma_{sb}-\gamma)$ identifies the existence region of the leaky wave inside a shadow boundary cone $\gamma=\gamma_{sb}$. In the same way, the existence function $U(\gamma_b-\gamma)$ defines a shadow boundary cone (SBC) at $\gamma=\gamma_b$ which bounds the existence region of the lateral wave.
Actually, in all the practical configurations we have investigated, the lateral wave contribution has systematically been found negligible due to prevalence of the space-wave contribution. Thus, the corresponding uniform asymptotics will not be presented in this paper.

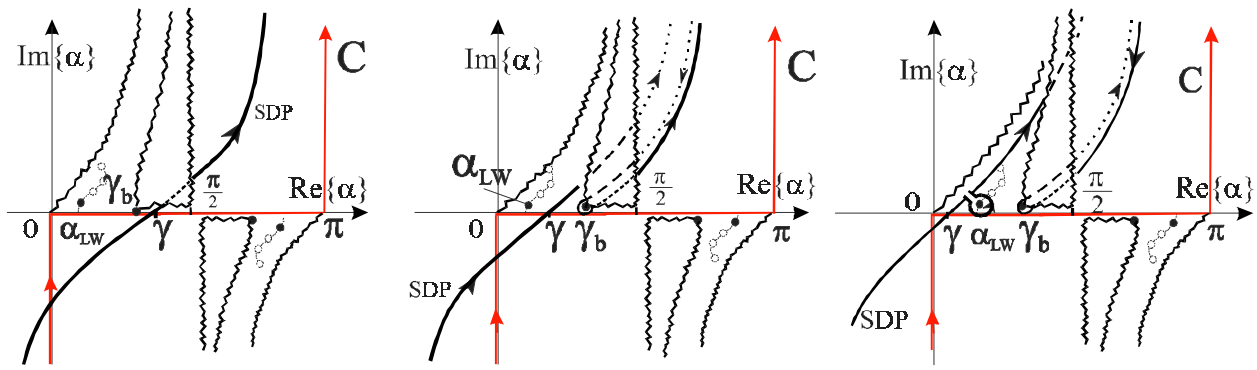


Fig. 1. *Angular spectral plane. The original contour C=($-j\infty$, 0, π, $\pi+j\infty$) is deformed into the SDP through the saddle point γ. Branch points (fig. b and c) and leaky wave pole (fig. c) are captured in the contour deformation. L-type branch point at γ_b maps the square-root branch cuts at k_1 in the k_x-plane. Other log-type branch cuts at γ_b and 0 occur, that correspond to the log-type branch cuts at k_1 and k_2 in the k_x-plane, respectively. The continuous line paths are on the top Rieman sheet (RS) associated with every branch cuts. The long dashed line paths are on top RS of the square-root type branch cut at γ_b and on bottom RS of the log-type branch cut at γ_b. Short dashed line paths are on bottom RS of the square-root type branch cut at γ_b and on top RS of the log-type branch cut at γ_b. Dotted line are on bottom RS of both type of branch cuts at γ_b.*

In (3), the leaky wave contribution is obtained from the residue of $f(\alpha)$ in (2) as

$$I_{LW} = \frac{-\sqrt{2j}}{\sqrt{r\pi \sin\gamma}\,2} \quad \frac{e^{-jk_2 r\cos(\alpha_{LW}-\gamma)}}{\sqrt{k_2 \sin\alpha_{LW}}\,D'(k_x^{LW})} \tag{7}$$

It is an evanescent cylindrical wave which propagates with a phase velocity $\omega/(k_2 \cosh(\alpha_{LW}^{im}))$ less than the wavespeed of the ambient medium, and along a direction which forms an angle α_{LW}^{re} with respect to the x axis. The last step in the uniform asymptotic evaluation is the analytical approximation of the integral along the SDP. The asymptotic evaluation is performed via the Van der Waerden (VdW) method [4] and the final result can be expressed as

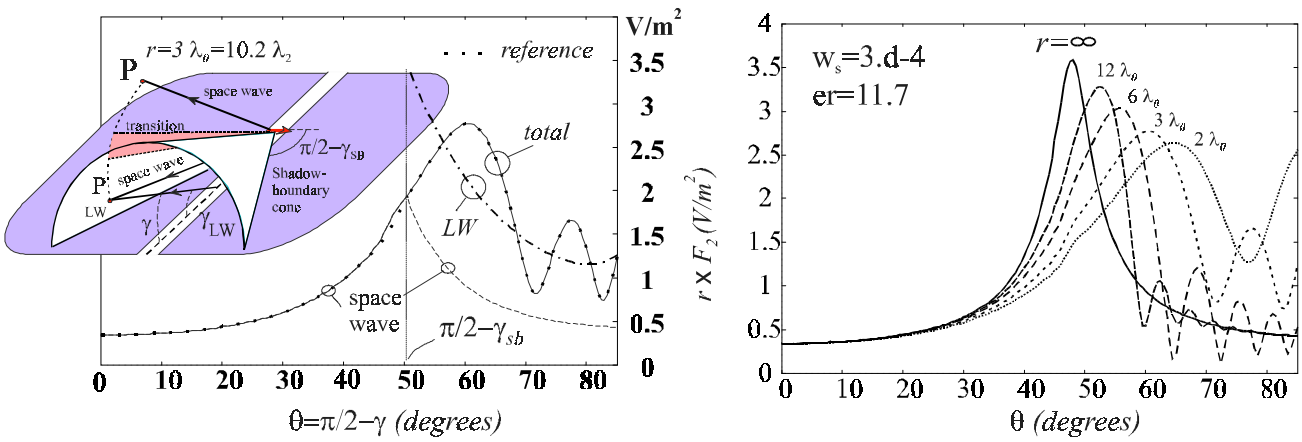
$$I_{sdp} \sim \tilde{I}_{sdp} = -\frac{e^{-jk_2 r}}{2\pi r}\left(\frac{1}{D(k_2\cos\gamma)} + \frac{(1-F[2k_2 r \sin^2(\frac{1}{2}(\alpha_{LW}-\gamma))]}{2k_2 \sin(\frac{1}{2}(\alpha_{LW}-\gamma))\sqrt{\sin\gamma \sin\alpha_{LW}}\,D'(k_x^{LW})}\right) \tag{8}$$

where $F(\xi)$ is the complex argument analytical continuation of the transition function of the Uniform Theory of Diffraction (UTD) [5].

NUMERICAL EXAMPLE AND CONCLUSION

The first set of curves in Fig. (2) check the accuracy of the asymptotic with respect to a reference solution obtained by numerical space domain integration of (6) in [2] (dots). In this figure, the normalized field potential is presented at r=3λ_0 versus the scan angle $\theta=90^o-\gamma$, for slots width $w_s=3\ 10^{-4}\lambda_0$ and dielectrics ϵ_{r1}=1 and ϵ_{r2}=11.7. The curve relevant to the leaky wave contribution alone is also presented (dash-dotted line), which is truncated at the shadow boundary. The

continuity of the total potential across the shadow boundary is guaranteed by the space wave (short dashed line). The sum of the LW and the SW uniformly recovers the reference solution. In the zone where the leaky wave exists one can observe that the total field presents amplitude oscillations due to the interference between leaky and space wave. A maximum of the total potential occurs at the first in-phase summation of this two contributions beyond the shadow boundary.



(a) ***(b)***

Fig. 2 *Electric potential F_2 in medium 2 at a distance $r = 3\lambda_0$ as a function of the scan angle for a slot with $w_s = 3\ 10^{-4}\lambda_0$ and for permitivities $\epsilon_{r1}=1$ and $\epsilon_{r2}=11.7$.* ***(a)*** *Comparison of reference curve (direct integration, dots) , Leaky wave (LW, dash-dotted line) and space wave (dashed line)*. ***(b)*** *Radial scans at different distances.*

To conclude a rigorous pole saddle point uniform asymptotics has been carried out starting from the continuous spectrum representation for the slot's Green's function solution. It is found that the conical leaky-wave is exponentially attenuated along radial scans in its existence region, and therefore disappears in the far zone. The directive pattern is then constructed by the sole space wave contribution, whose directive property are explained by the vicinity of the leaky wave pole to the visible portion of the space-wave spectrum. The validity and accuracy of the asymptotics has been verified by comparison with independent numerical integration, for observation points up to distance $0.5\ \lambda_n$, where λ_n (n=1,2) is the wavelength pertinent to the medium n=1,2.

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