NEAR-FIELD LINE-INTEGRAL REPRESENTATION OF THE KIRCHHOFF-TYPE APERTURE RADIATION FOR A PARABOLIC REFLECTOR

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Abstract

In this paper, a line integral representation is presented for a linearly polarized Kirchhoff-type aperture radiation from a parabolic reflector antenna. The main application of this result is concerned with the acceleration of the numerical integration for near field calculation of large reflector antennas. The present formulation is well suited to be improved by fringe diffraction contributions in the framework of edge-wave theories such as Physical Theory of Diffraction (PTD) and Incremental Theory of Diffraction (ITD).

I. Introduction

Reflector antenna analysis is typically performed by Physical Optics (PO); for simple shapes (i.e. parabolic reflectors), the combination of Geometrical Optics (GO) and Kirchhoff-type aperture integration (K-AI) provides an alternative scheme to evaluate near- to far-field contributions. In this framework a systematic procedure to reduce the aperture integration to a line integration along the rim may drastically improve the calculation time, particularly in the near field region, where FFT algorithms cannot be applied. In this paper a line-integral representation is presented for the field radiated by a parabolic reflector antenna, illuminated by a focal axial symmetric feed-pattern with Huygens' type polarization. This formulation provides a quite simple and physically appealing procedure to observe the field from near to intermediate region with a reduced numerical effort, with a practical utility when this field is used to illuminate independent objects placed in the vicinity (e.g. sub-reflector, struts, etc.). Generally speaking, this efficient representation assumes practical importance for problems of antenna analysis in presence of perturbing objects as well as problems of synthesis with near-field constrains. Furthermore, the algorithm is well suited to be improved by more sophisticated diffraction methods such as the Physical Theory of Diffraction (PTD), and the Incremental Theory of Diffraction (ITD) thus overcoming the near axis caustic problems of the Geometrical and Uniform Theory of Diffraction (GTD, UTD). The procedure presented here, while specializing to the case of reflector antennas the work presented in [1], leads to an interesting generalization to non-uniform aperture fields. In a first instance we assume an artificious feed pattern which imply a purely polarized plane wave on the aperture plane. For this case we rigorously derive the exact near-field radiated by the aperture in terms of line integration along the contour rim. The artificious assumption for the feed pattern is next removed and the formulation is properly corrected to account for the amplitude variation. The examples presented are concerned with a Gaussian perfectly polarized primary feed, which is a good approximation of the pattern from a circular corrugated horn. These examples show that even in the case of -30dB of edge tapering, the approximate line-integral formulation for non-uniform aperture field exhibits a surprising agreement when compared with a classical, time expensive, surface aperture near-field integration.

II. Geometry of the problem

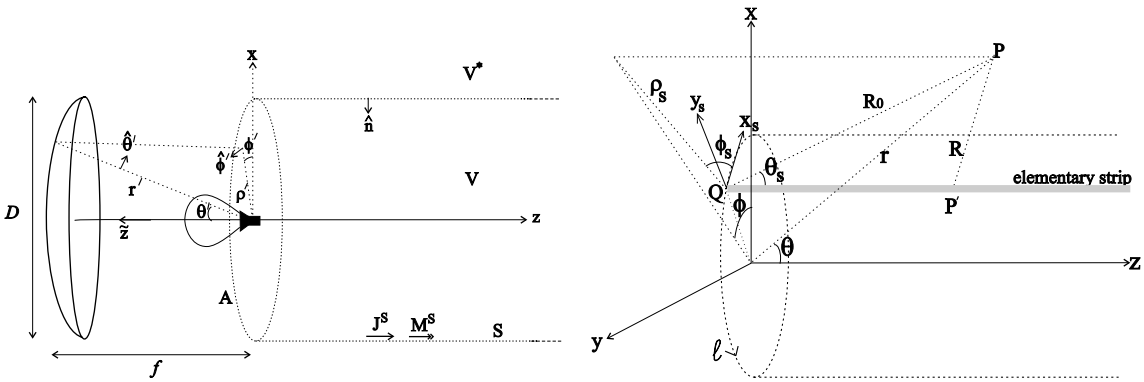
Consider a parabolic reflector with z-axis and diameter D, illuminated by a primary axial symmetric-field with phase center at the focus f (Fig.1a). Introduce a local reference system with $\tilde{z}$-axis pointing toward the vertex of the reflector and the spherical coordinates (r', θ', ϕ') with unit vectors, we express the primary field as

$$\mathbf{E}_{feed} = V_0 \frac{e^{-jkr'}}{4\pi r'} \mathbf{P}(\theta', \phi') F(\theta', \phi'), \tag{1}$$

where the polarimetric vectoral factor $\mathbf{P}(\theta', \phi') = (1 + \cos\theta')\ (\hat{\theta}' \sin\phi' + \hat{\phi}' \cos\phi')$ is the same as that of a Huygens' source placed at the focus and radiating towards the reflector; $F(\theta', \phi')$ is a positive normalized amplitude factor that approximates the feed radiation pattern. Using the geometric relations $r' = f\left[1 + (\rho'/2f)^2\right]$, $\theta' = 2\tan^{-1}(\rho'/2f)$ and $(1 + \cos\theta') = 2/\left[1 + (\rho'/2f)^2\right]$, the Geometrical Optics (GO) aperture field is

$$\mathbf{E}_a = -V_0 \frac{e^{-j2kf}}{4\pi} \frac{|\mathbf{P}(\theta', \phi')| F(\theta', \phi')}{f\left[1 + \left(\frac{\rho'}{2f}\right)^2\right]} \hat{x}, \qquad \mathbf{H}_a = \frac{1}{\zeta} \hat{z} \times \mathbf{E}_a. \tag{2}$$

Note that the aperture electric field is perfectly polarized along $\hat{x}$ which is a direct consequence of the Huygens' source type illumination.



(a) (b)

Fig. 1. *(a)Paraboloid illuminated by a focal feed, projecting surface and relevant reference systems. (b) Local reference systems associated to an elementary strip.*

III. Line-Integral formulation for uniform aperture distribution

Let us first assume a feed pattern $F(\theta', \phi') = 2/(1 + \cos\theta')^2$ which implies a rigorous uniform aperture distribution $\mathbf{E}_a = E_0 \hat{x}$, where $E_0 = V_0 \frac{e^{-j2kf}}{4\pi f}$. The aperture field is therefore a purely polarized plane wave. Applying for this case the procedure suggested in [1], we rigorously derive the field radiated by the aperture in terms of line integration along the contour rim. To this end, let us consider the surface $A + S$ where A is the aperture passing through the focus of the paraboloid and S is the lateral surface of a tube with circular cross-section and directrix parallel to the z-axis (Fig.1a). Denote by ℓ the contour of A, $\hat{n}$ the unit normal internal to S, and V and V^* the volumes inside and outside the semi-infinite tube, respectively (Fig.1a). The observation point P is described by rectangular coordinates (x, y, z) as well as by spherical coordinates (r, θ, ϕ). We now apply the

equivalence principle to the surface $A+S$ and to the plane wave field $(\mathbf{E}_a e^{-jkz}, \mathbf{H}_a e^{-jkz})$, which is the Maxwellian continuation of the aperture field. The above field may be regarded in V as the sum of the field radiated by the aperture and the field associated to an equivalent electric and magnetic current distribution located on S. The same total current contributions radiate a vanishing field in V^*. Therefore,

$$\mathbf{E}^A + \mathbf{E}^S = \mathbf{E}_a e^{-jkz} \quad \text{in } V, \quad \mathbf{E}^A + \mathbf{E}^S = 0 \quad \text{in } V^*. \tag{3}$$

Using the characteristic function $U(P)$ which is unity for $P \in V$ and zero for $P \in V^*$, we can rewrite (3) in a compact form as

$$\mathbf{E}^A = -\mathbf{E}^S + \mathbf{E}_a e^{-jkz} U(P), \quad \mathbf{H}^A = -\mathbf{H}^S + \mathbf{H}_a e^{-jkz} U(P). \tag{4}$$

Eq.(4) provides an alternative exact representation of the aperture field $\mathbf{E}^A$, with the advantage that the field $\mathbf{E}^S$ can be exactly represented as a line integral along the contour ℓ. The surface integration over the domain S can be divided into sequential line integrals along the rim contour ℓ and along the elementary strips. The integral along each strip is evaluated in exact analytical form by using the same procedure as that used in [2]; thus obtaining an exact line integral expression for $\mathbf{E}^S$ and $\mathbf{H}^S$:

$$\mathbf{E}^S = \int_\ell [\mathbf{e}_J(\ell) + \mathbf{e}_M(\ell)]\, d\ell, \quad \mathbf{H}^S = \int_\ell [\mathbf{h}_J(\ell) + \mathbf{h}_M(\ell)]\, d\ell, \tag{5}$$

where

$$\left\{ \begin{array}{c} \mathbf{e}_J(\ell) \\ \mathbf{h}_M(\ell) \end{array} \right\} = G(R_0) \left(-\frac{\hat{R}_0}{jkR_0} + \hat{\theta}_s \cot \frac{\theta_s}{2} \right) \left\{ \begin{array}{c} \zeta\, (\hat{n} \times \mathbf{H}_a) \cdot \hat{z} \\ \frac{1}{\zeta} (\mathbf{E}_a \times \hat{n}) \cdot \hat{z} \end{array} \right\}, \tag{6}$$

$$\left\{ \begin{array}{c} \mathbf{e}_M(\ell) \\ \mathbf{h}_J(\ell) \end{array} \right\} = \left\{ \begin{array}{c} -\hat{\phi}_s \\ \hat{\phi}_s \end{array} \right\} \cot \frac{\theta_s}{2}\, G(R_0) \left\{ \begin{array}{c} \mathbf{E}_a \times \hat{n}) \cdot \hat{z} \\ (\hat{n} \times \mathbf{H}_a) \cdot \hat{z} \end{array} \right\}, \tag{7}$$

and $(\hat{R}_0, \hat{\theta}_s, \hat{\phi}_s)$ are the unit vectors associated to local spherical coordinate system (R_0, θ_s, ϕ_s) associated to the end-point Q of each elementary strip (Fig.1b).

The structure of the incremental fields $(\mathbf{e}_J, \mathbf{h}_J)$ and $(\mathbf{e}_M, \mathbf{h}_M)$ resembles that of an equivalent pair of electric and magnetic dipoles $(\mathbf{J}^S, \mathbf{M}^S)$ directed along $\hat{z}$, whose amplitudes are proportional to the exciting aperture field at the rim. The pattern of these equivalent dipoles has a dominant asymptotic contribution given by $\cot \frac{\theta_s}{2}$ and exhibits a singularity for $\theta_S = 0$ (i.e., in the direction $\hat{z}$). This singular behavior of the integrand function results in a discontinuous behavior of the integral (i.e. of $(\mathbf{E}^S, \mathbf{H}^S)$) that smoothly and precisely compensates for the GO discontinuity when the observation point crosses the surfaces.

IV. Non-uniform aperture distribution

The exact procedure derived in section III is based on the construction of a rigorous Maxwellian field, which is obtained from the aperture field by a somewhat artificious assumption for the primary field. The above procedure, however, provides a useful guideline for the description to a non uniform, aperture field which is slowly varying with respect to the wavelength. To this end let us consider a "quasi-Maxwellian" field $(\widetilde{\mathbf{E}}_a e^{-jkz}, \widetilde{\mathbf{H}}_a e^{-jkz})$, which is constructed by forcing the curl Maxwell's equations to the tapered aperture GO field in (2) without significant restriction on $F(\theta', \phi')$:

$$\widetilde{\mathbf{E}}_a = \mathbf{E}_a - j\frac{\zeta}{k} \nabla_t \times \mathbf{H}_a, \quad \widetilde{\mathbf{H}}_a = \mathbf{H}_a + j\frac{1}{\zeta k} \nabla_t \times \mathbf{E}_a, \tag{8}$$

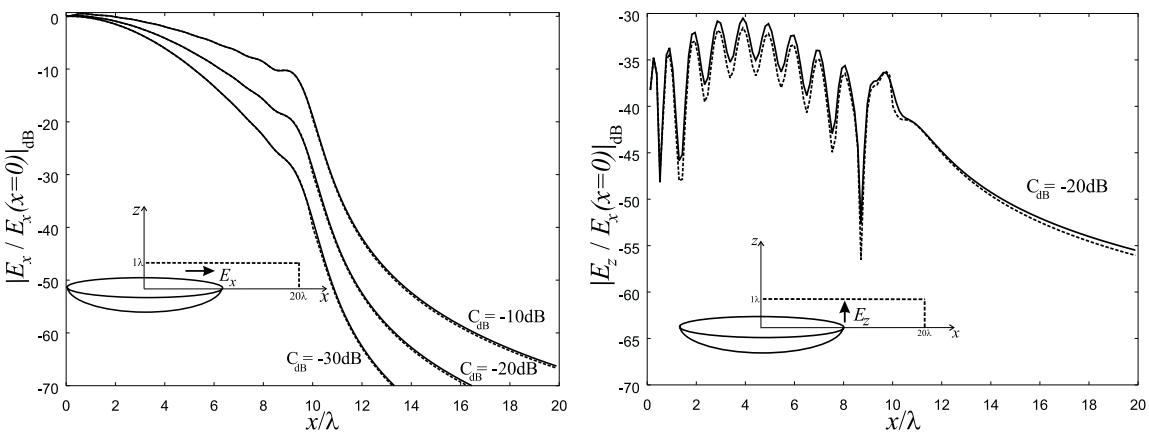
where $\nabla_t = \frac{\partial}{\partial x} \quad \hat{x} + \frac{\partial}{\partial y} \quad \hat{y}$ is the transverse gradient operator with respect to z.

Applying the process described in section III to the quasi-Maxwellian field in (8) leads to the following asymptotic approximation:

$$\left\{ \begin{array}{c} \mathbf{E}^A \\ \mathbf{H}^A \end{array} \right\} \simeq - \int_{\ell} \left\{ \begin{array}{c} \mathbf{e}_J + \mathbf{e}_M \\ \mathbf{h}_J + \mathbf{h}_M \end{array} \right\} d\ell + \left\{ \begin{array}{c} \widetilde{\mathbf{E}}_a \\ \widetilde{\mathbf{H}}_a \end{array} \right\} e^{-jkz} U(P). \tag{9}$$

In (9), $(\mathbf{e}_J, \mathbf{h}_J)$ and $(\mathbf{e}_M, \mathbf{h}_M)$ are obtained from (6)-(7), by replacing therein the aperture field with the modified expression of the aperture field in (8). The terms $\mathbf{E}'_a$ and $\mathbf{H}'_a$ provides a significant correction to the tapered aperture GO field, principally for the z-components.

Numerical results are presented hereafter for a normalized amplitude Gaussian primary field radiation with $F(\theta') = e^{-\theta'^2 c}$. Note that this field becomes a good approximation of that of a circular corrugated cross-polar compensated feed-horn. (The constant c can be conveniently written in terms of the edge illumination field's level C_{dB} (in dB) with respect to the maximum value trough $c = \frac{C_{dB}}{20\theta_0^2 \log e}$). The line-integral formulation is applied to a paraboloid reflector with $D = 20\lambda$ and $f = 10\lambda$. Reference result are obtained by conventional aperture integration of the electric and magnetic aperture currents. The presented curves show the x- and z-component of the aperture near field for different values of C_{dB}. For the very near zone, the result from our method agree very well with the reference solution obtained by conventional surface integration. It is worth pointing out that the tapered shape of the aperture field is increased by the Gaussian feed distribution and that gives rise to a z-component of the aperture field that does not appear in the case of a uniform aperture distribution.



(a) (b)

Fig. 2. *Near-field pattern at $\phi = 0$ and $z = 1\lambda$; (a) x-component of electric field for different Gaussian illuminations $C_{dB} = -10dB, -20dB, -30dB$, (b) z-component of electric field for $C_{dB} = -20dB$. Surface-integral (solid line); line-integral (dashed line).*

References

[1] A. Rubinowicz, "Geometric Derivation of the Miyamoto-Wolf Formula for the Vector Potential Associated with a Solution of the Helmholtz Equation", *J. Opt. Soc. Am.*, vol. 52, n. 6, pp. 717-718, June 1962.

[2] F. Mioc, M. Albani, P. Focardi, and S. Maci, "Line-Integral Representation of the Field Radiated by a Rectangular Waveguide Modal Current Distribution", *IEEE Trans. Antennas Propagat.*, Vol. 47, No. 2, pp. 408-410, Feb.1999.