UAPO DIFFRACTED FIELD BY ROUGH LOSSY WEDGES

P. Bernardi [1], R. Cicchetti [1], C. Gennarelli [2], G. Pelosi [3], G. Riccio [2]

[1] Università di Roma “La Sapienza”, via Eudossiana 18, 00184 Roma
[2] Università di Salerno, via Ponte Don Melillo, 84084 Fisciano (SA)
[3] Università di Firenze, via C.Lombroso 6/17, 50134 Firenze

Abstract

A Uniform Asymptotic Physical Optics solution, employed to evaluate the field diffracted by the edge in a rough lossy wedges, is presented. Such a solution is obtained by using a Physical Optics approximation of the surface current densities induced by an incident field on the illuminated face and by performing a uniform asymptotic evaluation of the resulting radiation integral. Its analytical expression, given in terms of the transition function of the Uniform Geometrical Theory of Diffraction, is easy to handle and to implement in a computer code. It can be used to evaluate the diffraction by buiding corners in urban environments.

1. INTRODUCTION

With the spreading of wireless networks for personal communication services, various models have been developed for simulating multipath propagation conditions in urban areas. The model that has emerged as being well suited to this end uses a ray-tracing approach based on the Uniform Theory of Diffraction (UTD) [1]. Ray-tracing approximates electromagnetic waves as discrete propagating rays that undergo scattering phenomena due to the presence of building, walls and other possible obstructions in the considered environment [2]-[5]. Accordingly, for an accurate prediction of the field propagation from the transmitter to the receiver, reflection from smooth and rough lossy surfaces and diffraction from edges must be properly taken into account in the UTD formulation. For calculating the fields around finite conductivity wedges, a heuristic version of UTD, including surface roughness effects, was developed by Luebbers in [2]. The corresponding solution for rough lossy wedges is easy to handle and to implement in computer programs, so that it is commonly used in ray-tracing propagation algorithms for wireless communication systems in urban areas. Despite the large use of such an approximate solution for solving practical problems, it must be remembered that it does not possess a rigorous basis, and therefore it should be used only with considerable attention.

A Uniform Asymptotic Physical Optics (UAPO) approach has been recently proposed in [6] to find the field diffracted from a nonpenetrable half-plane characterized by an anisotropic impedance boundary condition. The solution has been obtained by using a PO approximation of the surface current densities induced on the illuminated face of the scatterer and by performing a uniform asymptotic evaluation of the resulting radiation integral. Moreover, it must be stressed that the so obtained solution relies on a sound theoretical foundation, is easy to implement in efficient algorithms and, although approximate, is resulted to be accurate when compared with other more rigorous solutions available in literature.

In accordance with [6], the UAPO solution for the field diffracted from a rough lossy wedge, when illuminated by a normally impinging plane wave, is presented. Due to the lossy material forming the scatterer, the transmission through the wedge is neglected and the Fresnel’s reflection coefficients for parallel and perpendicular polarizations, properly corrected to take into account the roughness effects, are used in the expressions of the PO currents. Note that such a solution can be properly extended to consider the diffraction from obstacles in natural and urban environments, for which an exact solution is unknown.

2. UTD FORMULATION OF THE PROBLEM

Let us consider a rough lossy wedge surrounded by free-space. The z-axis of a cylindrical reference system is directed along its edge and the angle ϕ' denotes the propagation direction of a plane wave normally impinging on it (see Fig. 1). The exterior angle is equal to $n\pi$. The observation point P external to the wedge is identified by (ρ, ϕ).

According to the UTD formulation, the total electric field at P is expressed as sum of the Geometrical Optics (GO) field ($\boldsymbol{E}^{go}$) and the field ($\boldsymbol{E}^{d}$) diffracted by the edge. As well known, the GO field is given as superposition of the incident field and the field reflected from S_0 and S_n (the wedge surfaces at $\phi = 0$ and $\phi = n\pi$, respectively), namely,

$$\boldsymbol{E}^{go} = \boldsymbol{E}^{i}\, U^{i} + \boldsymbol{E}_0^{r}\, U_0^{r} + \boldsymbol{E}_n^{r}\, U_n^{r} \tag{1}$$

where U^i, U_0^r and U_n^r are unit step functions equal to 1 in the regions illuminated by the incident and reflected fields and 0 in their shadow regions. For what concerns the evaluation of the field reflected from the wedge surfaces, it is convenient to introduce the diagonal reflection matrices $\underline{\boldsymbol{R}}^0$ and $\underline{\boldsymbol{R}}^n$ associated with the ordinary incidence planes for S_0 and S_n. Their elements relate the field reflected from an equivalent infinite rough lossy surface to the incident field. In particular, with reference to S_0 (see Fig. 2), it results:

$$\boldsymbol{E}_0^{r} = \begin{pmatrix} E_{\parallel 0}^{r} \\ E_{\perp 0}^{r} \end{pmatrix} \mathrm{e}^{jk\rho\cos(\phi+\phi')} = \underline{\boldsymbol{R}}^0 \begin{pmatrix} E_{\parallel 0}^{i} \\ E_{\perp 0}^{i} \end{pmatrix} \mathrm{e}^{jk\rho\cos(\phi+\phi')} = \begin{pmatrix} R_{\parallel}^{0} & 0 \\ 0 & R_{\perp}^{0} \end{pmatrix} \begin{pmatrix} E_{\parallel 0}^{i} \\ E_{\perp 0}^{i} \end{pmatrix} \mathrm{e}^{jk\rho\cos(\phi+\phi')} \tag{2}$$

wherein k is the free-space wavenumber, the notations $||$ and $\perp$ denote the field components (at the origin) parallel and perpendicular to the ordinary plane of incidence (see Fig. 2). The reflection coefficients for parallel and perpendicular polarizations, determined by material properties, incidence direction and frequency, are given by [4], [7]:

$$R_{\parallel}^{0} = \rho_s \frac{\varepsilon_{\mathrm{r}} \sin\phi' - \sqrt{\varepsilon_{\mathrm{r}} - \cos^2\phi'}}{\varepsilon_{\mathrm{r}} \sin\phi' + \sqrt{\varepsilon_{\mathrm{r}} - \cos^2\phi'}} \quad ; \qquad R_{\perp}^{0} = \rho_s \frac{\sin\phi' - \sqrt{\varepsilon_{\mathrm{r}} - \cos^2\phi'}}{\sin\phi' + \sqrt{\varepsilon_{\mathrm{r}} - \cos^2\phi'}} \tag{3}$$

where ε_r is the complex relative permittivity of the wedge structure and ρ_s is the surface roughness attenuation factor. By using the Gaussian model for a rough surface, ρ_s can be derived by the expressions [4]

$$\rho_s = \mathrm{e}^{-\delta} \quad ; \qquad \delta = 8\left(\pi \frac{\Delta h}{\lambda} \sin\phi'\right)^2 \tag{4}$$

where λ is the wavelength and Δh is the standard deviation of the normal distribution of heights.

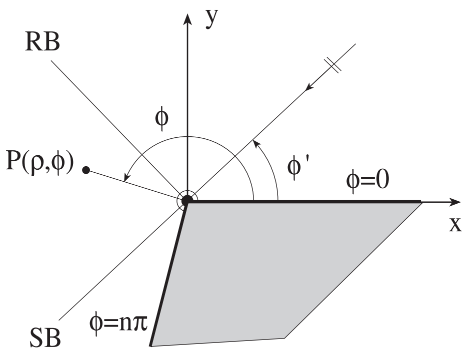

Fig. 1. Geometry of the problem.

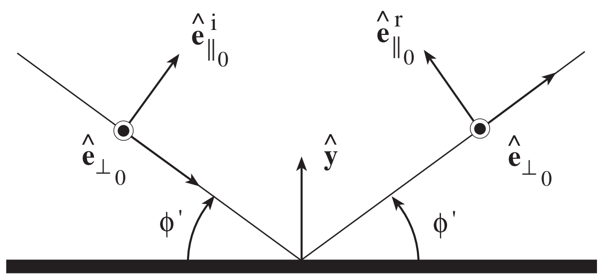

Fig. 2. Geometry for the evaluation of the reflection coefficients for S_0.

3. DIFFRACTED FIELD: UAPO SOLUTION

The field generated by the PO electric and magnetic surface currents $\boldsymbol{J}_s^{PO}$ and $\boldsymbol{J}_{ms}^{PO}$

induced by the incident field on the wedges surfaces can be expressed in the far-field region by means of the well known radiation integral:

$$\boldsymbol{E}^{PO}=\boldsymbol{E}_0^{PO}+\boldsymbol{E}_n^{PO}\cong -jk\iint\limits_{S}\left[\left(\underline{\boldsymbol{I}}-\hat{\boldsymbol{R}}\hat{\boldsymbol{R}}\right)\cdot\left(\zeta\boldsymbol{J}_s^{PO}\right)+\boldsymbol{J}_{ms}^{PO}\times\hat{\boldsymbol{R}}\right]\frac{e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}S \tag{5}$$

where $S=S_0+S_n$. In (5) ζ is the free-space impedance, $\boldsymbol{r}$ and $\boldsymbol{r}'$ denote the observation and source points, respectively, $\hat{\boldsymbol{R}}$ is the unit vector from the radiating element at $\boldsymbol{r}'$ to the observation point and $\underline{\boldsymbol{I}}$ is the (3×3) identity matrix. Since the two field contributions can be determined in similar way, only $\boldsymbol{E}_0^{PO}$ is explicitly considered. By following the approach in [6], it results:

$$\boldsymbol{E}_0^{PO}\cong\left[\left(\underline{\boldsymbol{I}}-\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}\right)\cdot\left(\zeta\tilde{\boldsymbol{J}}_{s_0}^{PO}\right)+\tilde{\boldsymbol{J}}_{ms_0}^{PO}\times\hat{\boldsymbol{s}}\right]\frac{1}{4\pi j}\int_{(c)}\frac{e^{-jk\rho\cos(\alpha+q_0\phi)}}{\cos\alpha+\cos\phi'}\,\mathrm{d}\alpha \tag{6}$$

wherein $\hat{\boldsymbol{s}}$ identifies the diffraction direction,

$$q_0=\begin{cases}-1 & \text{for}\quad 0<\phi<\pi\\ \ \ 1 & \text{for}\quad \pi<\phi<n\pi\end{cases} \tag{7}$$

$$\zeta\tilde{\boldsymbol{J}}_{s_0}^{PO}=\left(1-R_\perp^0\right)E_{\perp 0}^i\,\sin\phi'\,\hat{\boldsymbol{e}}_{\perp 0}+\left(1+R_{\|}^0\right)E_{\|_0}^i\left(\hat{\boldsymbol{y}}\times\hat{\boldsymbol{e}}_{\perp 0}\right) \tag{8}$$

$$\tilde{\boldsymbol{J}}_{ms_0}^{PO}=\left(1-R_{\|}^0\right)E_{\|_0}^i\,\sin\phi'\,\hat{\boldsymbol{e}}_{\perp 0}-\left(1+R_\perp^0\right)E_{\perp 0}^i\left(\hat{\boldsymbol{y}}\times\hat{\boldsymbol{e}}_{\perp 0}\right) \tag{9}$$

By applying the Cauchy's theorem, the contribution to the PO field related to the integral along the contour (c) (distorted for the presence of singularities in the integrand) is equivalent to the summation of the integral along the Steepest Descent Path (SDP) passing through the pertinent saddle-point α_s and the residue contributions associated with all those poles α_p that are inside the closed path (c)+SDP. As in [6], a uniform asymptotic evaluation of the integral along SDP provides the edge diffraction contribution in terms of the UTD transition function. The diffraction contribution related to S_n can be determined in a similar way, so that the UAPO solution for the diffracted field can be written in the following compact form:

$$\boldsymbol{E}^d=\boldsymbol{E}_0^d+\boldsymbol{E}_n^d=\begin{pmatrix}E_\beta^d\\ E_\phi^d\end{pmatrix}=\underline{\boldsymbol{D}}\begin{pmatrix}E_{\beta'}^i\\ E_{\phi'}^i\end{pmatrix}\frac{e^{-jk\rho}}{\sqrt{\rho}} \tag{10}$$

where

$$\underline{\boldsymbol{D}}=\frac{-e^{-j\pi/4}}{2\sqrt{2\pi k}}\left[\underline{\boldsymbol{M}}^0\,\frac{F\left(k\rho\,a(\phi-q_0\phi')\right)}{\cos\phi+\cos\phi'}\,U_0+\underline{\boldsymbol{M}}^n\,\frac{F\left(k\rho\,a(2qn\pi-q_n\phi-\phi')\right)}{\cos(n\pi-\phi)+\cos(n\pi-\phi')}\,U_n\right] \tag{11}$$

In (11), U_0 and U_n are unit step functions which are equal to 1 or 0 depending on the fact that the corresponding face of the wedge is illuminated or not by the incident field, *F(x)* is the UTD transition function [1], $a(x)=2\cos^2(x/2)$ and

$$q=\begin{cases}0 & \text{for}\quad 0<\phi<(n-1)\pi\\ 1 & \text{for}\quad (n-1)\pi<\phi<n\pi\end{cases};\qquad q_n=\begin{cases}-1 & \text{for}\quad 0<\phi<(n-1)\pi\\ \ \ 1 & \text{for}\quad (n-1)\pi<\phi<n\pi\end{cases} \tag{12}$$

The matrices $\underline{\boldsymbol{M}}^0$ and $\underline{\boldsymbol{M}}^n$, which account for the expressions of the PO currents on the wedge surfaces, have been determined by following [6].

4. NUMERICAL RESULTS

Many numerical tests have been performed to verify the effectiveness of the UAPO solution for determining the field diffracted from rough lossy wedges illuminated at normal incidence by an electromagnetic plane wave. However, to save spacing, only the results relevant to one case are here reported. Figure 3 refers to a 90° rough lossy wedge characterized by $\varepsilon_r = \tilde{\varepsilon}_r - j60\sigma\lambda = 4.44 - j0.045$ and $\Delta h = 0.005$ m (parameters of a brick wall at the frequency $f = 4$ GHz [4]). The incidence direction is $\phi' = 70°$. The incident field is characterized by $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$ and the scattered field is evaluated at $\rho = 0.75$ m. As expected, the UAPO diffracted field compensates for the discontinuities of the GO field, so that the total field is continuous throughout, including across shadow and reflection boundaries, demonstrating the uniformity of the solution.

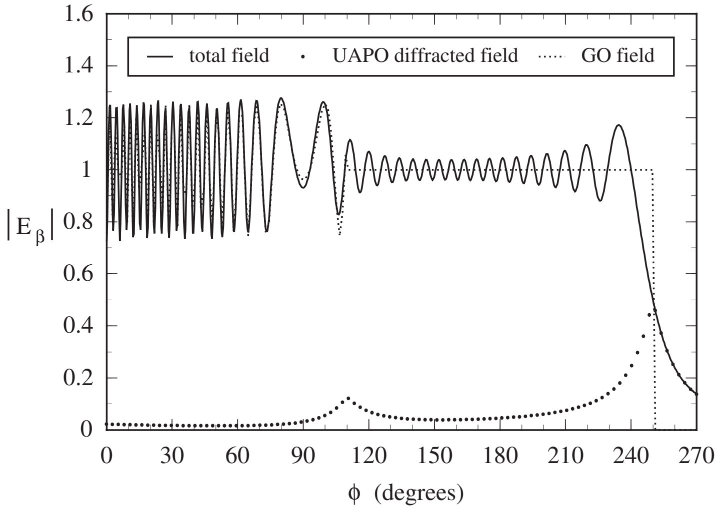

Fig. 3. Magnitude of the field scattered from a right-angle rough lossy wedge illuminated at $\phi' = 70°$. The observation point is at $\rho = 0.75$ m. The standard deviation of the normal distribution of heights is $\Delta h = 0.005$ m. The relative permittivity is $\varepsilon_r = 4.44 - j0.045$ at $f = 4$ GHz. Incident electric field: $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$.

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