A novel approach to shape reconstruction from field data

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Abstract- A new shape estimation technique, based on the matching between the theoretical Local Spectral Content of the reduced field and the one evaluated from the observed data, is presented. The approach, applied in the case of perfectly conducting scatterers, does not ask for the solution of the full inverse problem. The numerical analysis confirms the effectiveness of the technique.

1 Introduction

The solution of full Inverse Scattering (IS) problems in Electromagnetics and Acoustics [1] requires to find, from the field data collected on an observation domain, the position, the geometry and the electromagnetic properties of the scatterers.

However, in many practical applications, concise information about the radiating system can be of real interest either as the desired final result of the inversion process or as the result of a preprocessing step aiming to improve the reliability and to reduce the complexity of the IS algorithms. In particular, the concise information of interest are of geometrical kind as, f.i., the region containing the system (the minimum sphere [2], the convex hull [3], the shape [4, 5]) or the number and the locations of the observed objects [6, 7].

Obviously, such techniques are of practical interest if they can search for the few variables of interest without solving the full IS problem and disregarding the other parameters. This purpose can be successfully pursued by unconventional approaches exploiting observable properties of the field data that depends "only" on the geometry of the system.

The aim of this communication is to present a new approach to the shape reconstruction of scattering systems from field data at a single frequency. The method will be presented by referring to the two-dimensional geometry and to metallic scatterers in the electromagnetic case, even if it could be applied to acoustics and extended to the 3D case.

The technique exploits a new property of the electromagnetic field, the Point Source Spectral Content (PSSC), representing the local spectral content of the reduced field on the observation domain due to an elementary radiator and depending only on the relative position of the observation curve, the radiator and the reference point used in the definition of the reduced field [7].

The goal of the searching procedure is attained by looking for that shape whose Local Spectral Content (LSC), theoretically foreseen by superimposing the contributions of each elementary radiator, matches at best the one extracted from the observed field.

To get a successful reconstruction, a suitable strategy is adopted to enhance at best the information content on the geometry of the radiating system incorporated in the LSC. Numerical results define the performances of the technique.

2 Statement of the problem and solution strategy

With reference to a 2D geometry, let us consider the extended scatterer *S* embedded in an homogeneous medium with known electromagnetic parameters and the Cartesian reference system Σ (O,x,y) depicted in Fig. 1. We assume that the scatterer is illuminated by a set of n radiators, say $\mathcal{R}_1 \ldots \mathcal{R}_n$, and that the electromagnetic field **E** is collected on the observation curve Γ with equation y=d.

As mentioned in the introducing Section, the aim of the paper is to present a new approach to the problem of estimating the shape of the scatterer from field data at a single frequency without solving the full inverse problem (the approach could be easily extended to the multifrequency case).

To this end, we exploit two general properties of the reduced field, the PSSC and the LSC, introduced in [7], and whose definitions and characteristics are briefly summarized in the following.

Given an elementary radiator Q and a reference point R with vector position **r**' and $\mathbf{r}_0$, respectively, and an observation point P(ξ) on Γ, being ξ=x/λ and λ the wavelength, the "reduced" field is defined as $\mathbf{F}(\xi)=\exp(j\beta|\mathbf{r}(\xi)-\mathbf{r}_0|)\mathbf{E}(\xi)$, where the exp(jωt) time dependence is assumed and β=2π/λ.

By taking the sliding-window Fourier transform of **F** we get:

$$\widetilde{\mathbf{F}}(\xi,\mathbf{r}_0,\kappa,\mathbf{r}') = \int g(\xi'-\xi)\mathbf{F}(\xi')e^{-j\kappa\xi'}d\xi' = \int g(\xi'-\xi)e^{j\psi(\mathbf{r}_0,\xi',\mathbf{r}')}\mathbf{N}(\xi',\mathbf{r}')e^{-j\kappa\xi'}d\xi' \qquad (1)$$

where g is the windowing function, $\mathbf{N}(\xi',\mathbf{r}')$ is a slow varying vector term related to the Green function, κ is the spatial frequency and $\psi(\mathbf{r}_0,\xi',\mathbf{r}')=\beta(|\mathbf{r}(\xi')-\mathbf{r}_0|-|\mathbf{r}(\xi')-\mathbf{r}'|)$.
For $\beta\rightarrow\infty$, $\widetilde{\mathbf{F}}$ can be asymptotically evaluated by the stationary phase method. Neglecting end points and/or slope contributions, only the stationary points, which are the roots of the equation: $-\kappa+\partial\psi/\partial\xi'=0$, give a significant contribution to (1), provided that they are in the sliding window.

Accordingly, the sliding-window spectrum of $\mathbf{F}(\xi)$ is confined in a neighbourhood of (ξ, h(ξ, **r**')) where h(ξ, **r**') is the PSSC defined as [7]: $h(\xi,\mathbf{r}')=\partial\psi/\partial\xi$.

As long as a continuous or a discrete distribution of elementary radiating elements are considered, the LSC accounts for every elementary contribution.

As an example, Fig.2 shows the sliding-window Fourier transform of the reduced (with respect to the point (-50λ, -60λ)) field on the observation line y=50λ radiated by 4 point-like sources located at (-40λ, -60λ), (40λ, -60λ), (0, -20λ), (0, -80λ) and excited by currents equal to 0.803, 1, 0.962, 0.993, respectively. The theoretical PSSC's corresponding to each source are also presented as white curves. As seen, the contributions of each component are clearly distinguishable.

Accordingly, a technique searching for radiator locations [7] or the convex hull of radiating systems [3] can be obtained by matching the LSC estimated from the available field data to the theoretical one, corresponding to trial source distributions. In principle, also when dealing with extended continuous scatterers, the same approach could be used to retrieve the shape of the observed objects.

However, when a continuous distribution of radiating currents is considered, as in the case of the currents induced on an extended metallic scatterer, the contributions to the LSC due to contiguous elementary radiating elements can cancel each other out. In this case, the information content of the sliced Fourier Transform of the reduced field

involves both the geometry of the scatterer and the intensity of the scattering currents. As an example, let us consider an observation line y=40λ and a perfectly conducting scatterer with a contour described by the polar equation $\rho(\theta)=(5+15\cos^2(\theta))\lambda$ and illuminated by a point-like source located at (60λ,40λ). The Local Fourier transform of the scattered field is depicted in Fig.3 together with the theoretical PSSC's associated to the points on the contour of *S*. The plot clearly shows that a portion of the LCS, forecast by the theory, disappears, due to destructive interference between elementary contributions.

Accordingly, the shape reconstruction technique based on the above approach can fail to find the correct result. To circumvent this drawback and get an effective retrieving technique, a proper strategy must be devised.

The key point involves the role of the phase relationships between the elementary current contributions on the LSC estimated from the scattered field data. As a matter of fact, the particular relationship may determine the cancellation between the contributions to portions of the LSC due to the elementary currents. Accordingly, the proposed approach exploits a strategy able to avoid the effect of the interferences, by randomising the illumination. To this end, an ensemble of scattering experiences is obtained by randomly varying the excitations of the primary sources $\mathcal{R}_1 \ldots \mathcal{R}_n$. Then, the corresponding sliding-window Fourier Transform are evaluated and the LSC, to be compared with the theoretical one, is estimated from the mean of their amplitudes.

To show the effectiveness of the strategy, the scatterer previously considered has been illuminated by 14 elementary sources and an ensemble of 10 random illuminations has been considered. The LSC estimated as described above is depicted in Fig.4 together with the theoretical forecast (white lines).

A shape estimation technique, based on an evolutionary algorithm and exploiting the above-mentioned matching approach, has been implemented. The initial guess and the retrieved shape (stars) are shown in Fig.5 and 6, respectively, together with the scatterer shape (circles).

References

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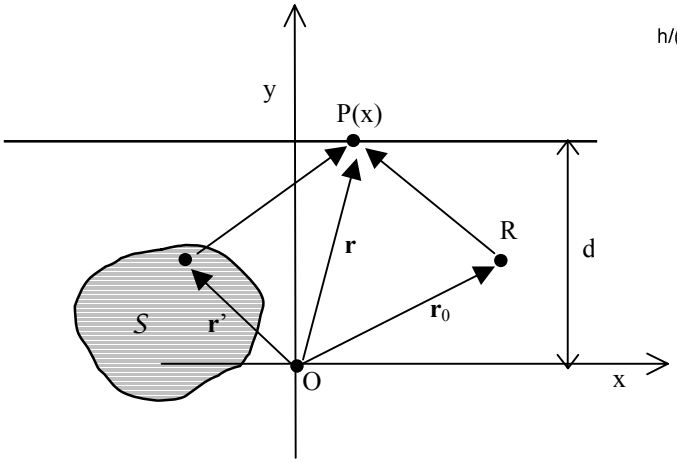

Fig.1 Geometry of the problem

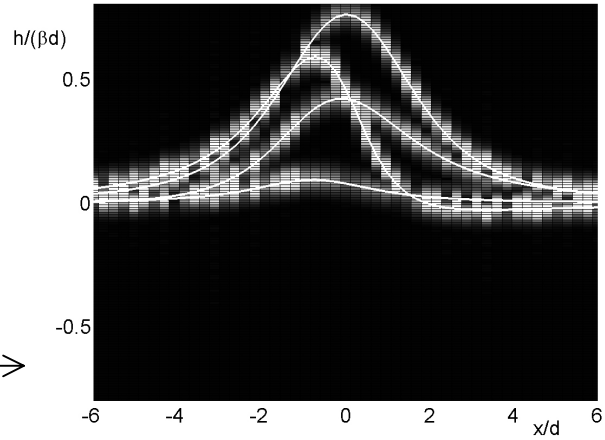

Fig. 2 Sliding-window Fourier transform: the case of 4 sources.

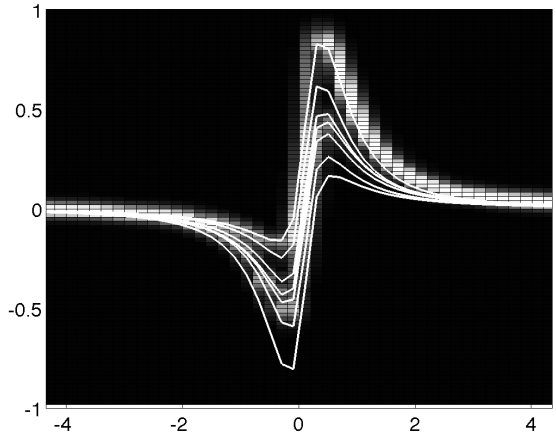

Fig. 3 Sliding-window Fourier transform: the scattering case

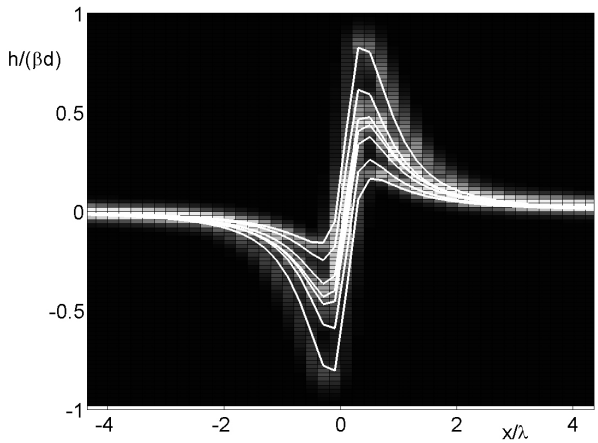

Fig.4 Sliding-window Fourier transform: the scattering case with randomised illumination contribution.

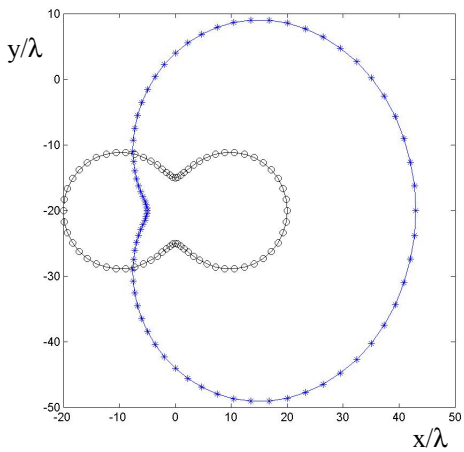

Fig. 5. Initial guess

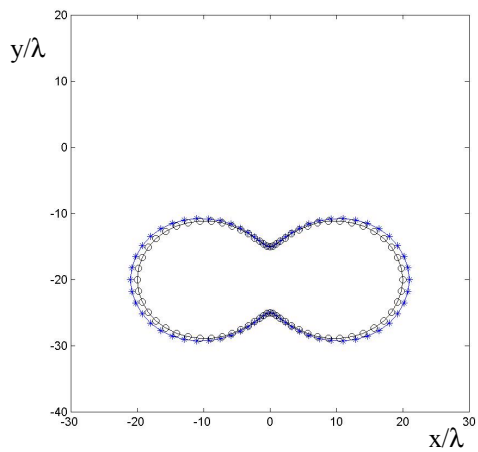

Fig. 6. Final estimate.