XIV RiNEM
Ancona, Sept. 16-19, 2002
Special Opening Session

Ray Chaos, Full Wave Oddities and Regular to Irregular Transition in EM Scattering from Complex Systems

I.M. Pinto, WavesGroup, University of Sannio at Benevento

The WavesGroup
http://www.thewavesgroup.unisannio.it

Università degli Studi del Sannio

Session Outline

XIV RiNEM
Ancona, Sept. 16-19, 2002
Special Opening Session

Ray Chaos, Full Wave Oddities
and Regular to Irregular Transition
in EM Scattering from Complex Systems

Talk 1

Chaos in a nutshell

Chaos in EMtics

Why studying ray-chaotic EM b.v.p.'s

Talk 2:

A new ray-chaotic EM b.v.p. paradigm,
Full-wave oddities
Regular-to-irregular transition

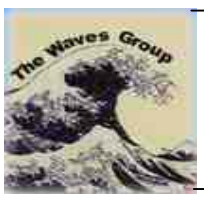
XIV RiNEM
Ancona, Sept. 16-19, 2002

Special Opening Session: Ray Chaos, Full Wave Oddities and Regular to Irregular Transition in EM Scattering from Complex Systems

Why Studying Ray Chaotic EM b.v.p.'s: an Introduction

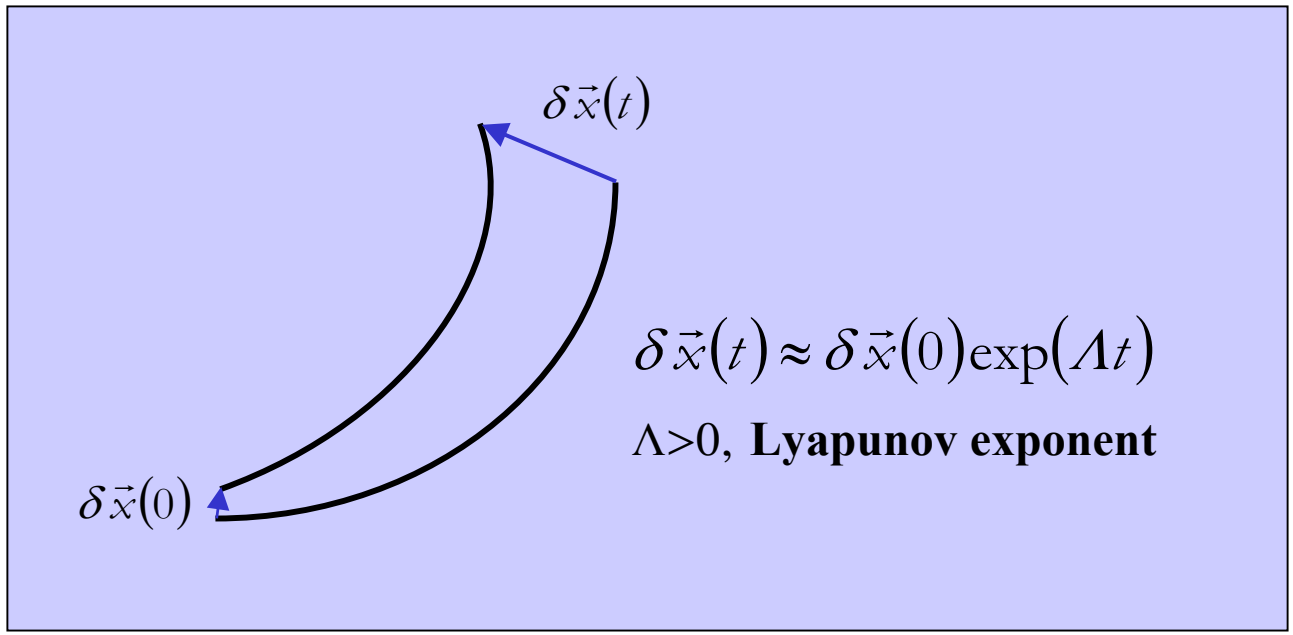
Innocenzo M. Pinto

WavesGroup, University of Sannio at Benevento, IT



The WavesGroup
http://www.thewavesgroup.unisannio.it

Deterministic Chaos in a Nuthell



$$\delta\vec{x}(t) \approx \delta\vec{x}(0)\exp(\Lambda t)$$

$\Lambda>0$, **Lyapunov exponent**

Exponential-in-time divergence of evolutions originating from nearby initial states.

Chaos Paradigms

$$x_{n+1} = \gamma x_n \left(x_{n+1} \right)$$ **Logistic map;**

$$\begin{cases} \dot{x} = \sigma \left(y - x \right) \\ \dot{y} = \rho x - y - xz \\ \dot{z} = xy - \beta z \end{cases}$$

Lorenz thermally-induced fluid convection;

$$\ddot{x} + kx + x^3 = B\cos\left(t\right)$$ **Duffing's equation.**

A Few Chaos Facts

- At any fixed precision, information on the initial state is ***steadily lost***. Measurements made on the system ***do not*** allow to predict its far-future state.

- Integrable systems (including ***all*** linear ones) ***can-not*** display chaos (positive λ);

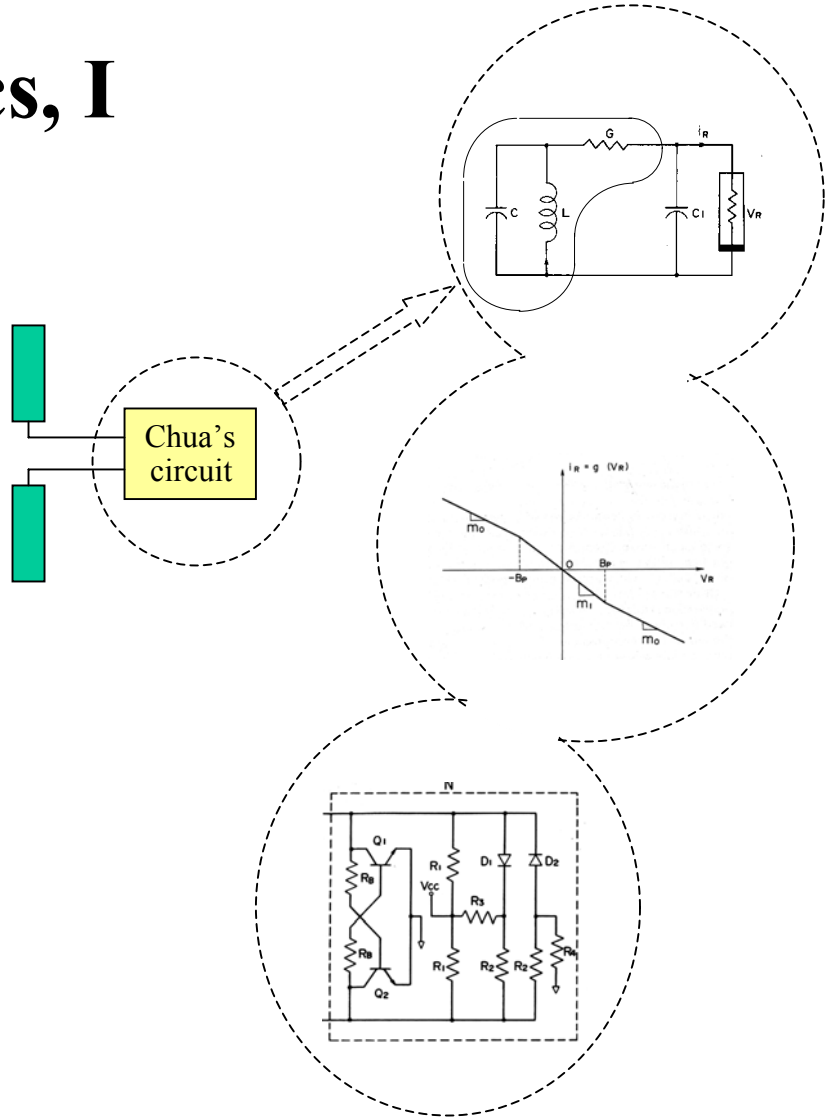
- Sufficiently many degrees of freedom required for chaos (2 for autonomous systems).

[E. Ott, *Chaos in Dynamical Systems*, Cambridge Univ. Press, 1993]

Chaos in Electromagnetics, I

A short antenna (actually, a loop) loaded by Chua's chaotic circuit.

[L.O. Chua, M. Komuro and T. Matsumoto, *The Double-Scroll Family*, IEEE Trans. **CAS-33**, 1073, 1986;
T. Matsumoto, L.O. Chua and a. Tokumasu, *Double Scroll via a two-Transistor Circuit*, IEEE Trans. **CAS-33**, 828, 1986]

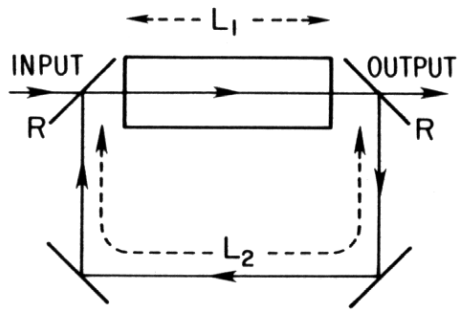


Chaos in Electromagnetics, II

A **variety of** problems involving (dispersive) **nonlinear constitutive properties** ⇨ nonlinear coupling among a set of modes.

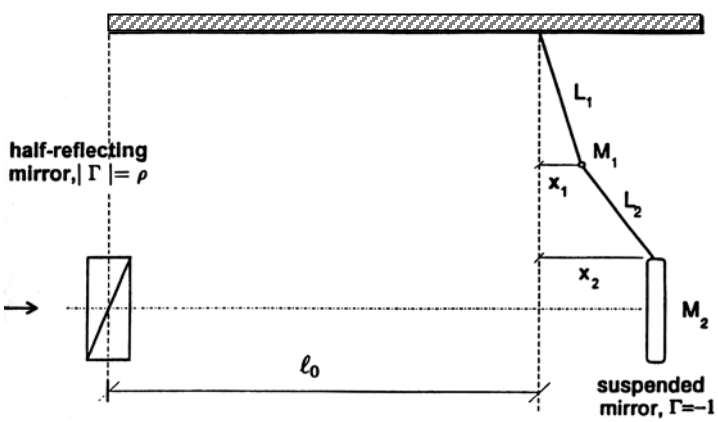
[see, e.g., N.B. Abraham, E. Arimondo, R.W. Boyd, *Instability, Dynamics and Chaos in Nonlinear Optical Systems*, in *Instability and Chaos in Quantum Optics*, N.B. Abraham, F.T. Arecchi and L.A. Lugiato, Eds., NATO-ASI B177, Plenum Press, NY, 1987]

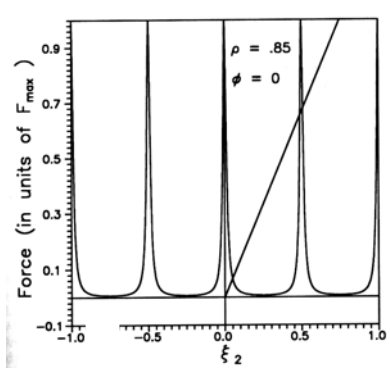
e.g., Ikeda's optically pumped ring.
[K. Ikeda, Opt. Comm., **30**, 257, 1979]

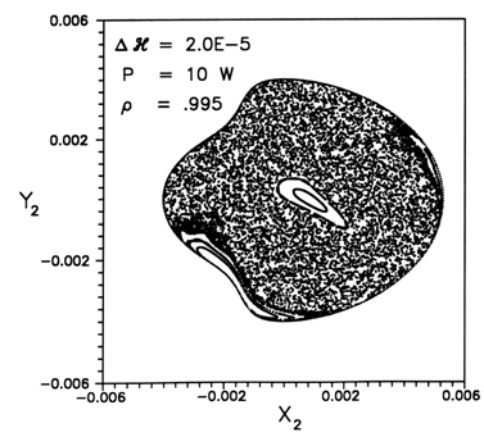


Chaos in Electromagnetics, III

Resonators involving elastically bound mirrors, where ***mechanical and EM*** degrees of freedom are coupled by radiation-pressure (everything else being ***linear***).





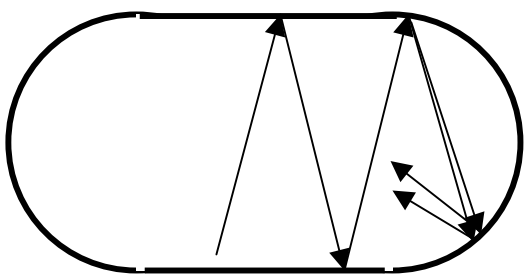


[V.Pierro and I.M Pinto, *Radiation Pressure induced Chaos in Multipendular Fabry Perot*, Phys. Lett, **A185**, 14, 1994; ibid., **A193**, 498, 1994.]

Chaos in Electromagnetics, IV

Boundary value problems featuring ***ray-chaos*** (exponential divergence between near-co-incident rays)…

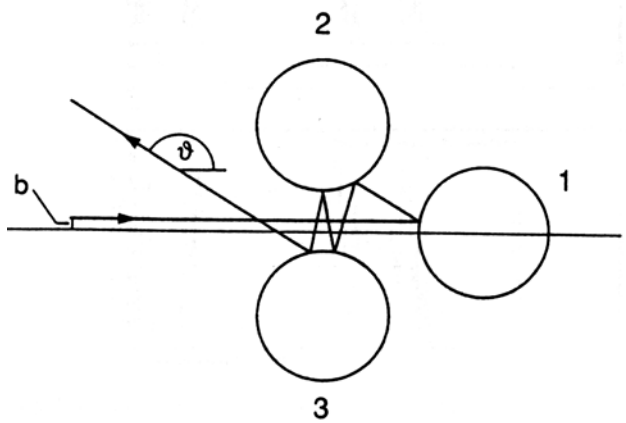
Internal b.v.p.'s



Bunimovic PEC "stadium" resonator

[S.W. McDonald and A.N. Kaufman, "*Wave Chaos in the Stadium: Statistial Properties of Short-Wave Solutions of the Helmholtz Equation*", Phys. Rev. **A37**, 3067, 1988.]

External b.v.p.'s

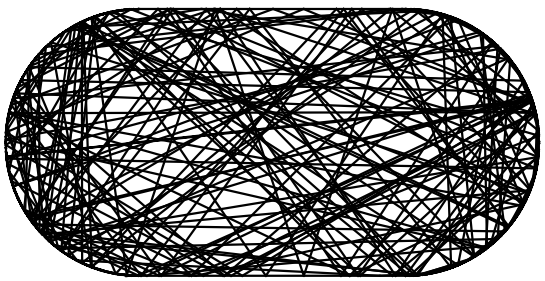


3-disk PEC "pinball" scatterer

[P. Gaspard and S.A. Rice, "*Scattering from a Classical Chaotic Repellor*", J. Chem. Phys., **90**, 2225, 1989; ibid., 2242; ibid., 2255]

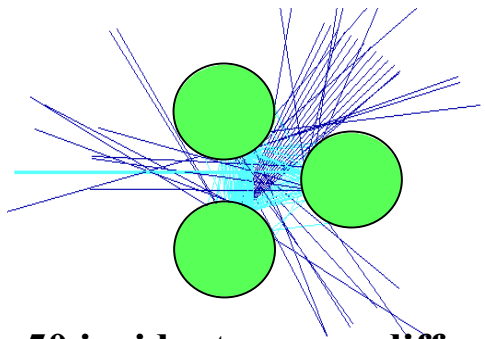
Chaos in Electromagnetics, IV, cont.d

Internal b.v.p.'s



...after 250 bounces

External b.v.p.'s



…50 incident rays w. different *b*

...where is nonlinearity ?

…in the ***maps*** evolving successive impact points

[MV: Berry, Eur. J. Phys., **2**, 91, 1981; P. Gaspard And S.A. Rice, J. Chem. Phys. 90, 2242, 1989]

Why studying R-chaotic EM BVP's

1. Because they provide *a viable* (!) *route* to understand the properties of generic (and otherwise untractable) ***complex / irregular*** b.v.p.'s - at least under suitable circumstances (extreme diffusion limit);

2. Because they pose a ***fundamental issue of principle*** about the hallmarks (if any) of ray - chaos in the short but full-wave regime, where the diffraction limit forbids the existence of structure at all scales below λ.

3. Because some of their ***unique*** *properties* might open the door to the design of *new devices* and *new artificially-engineered materials*;

1.

Taming Ergodicity and Localization In Complex/Irregular EM b.v.p.'s

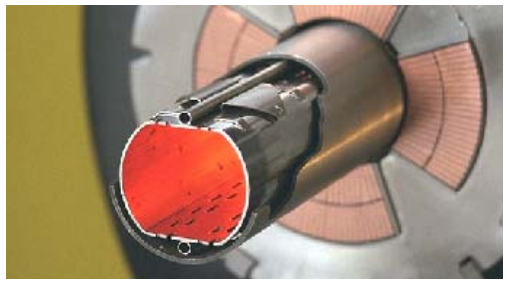
Complex Scatterers

"…consider for example the RCS of a target such as a B-52 aircraft at a frequency of 10GHz, where the target size exceeds 10^3 wavelengths in linear dimensions. The aspect-angle variation in the RCS of such a target is *extremely spiky*, with changes of 10s of dB possible for incidence angle changes of a few tenths of a degree." [E.K. Miller and T.K. Sarkar, in *Frontiers in Electromagnetics*, D.H. Werner and R. Mittra Eds, IEEE Press, 2000, p. 972]

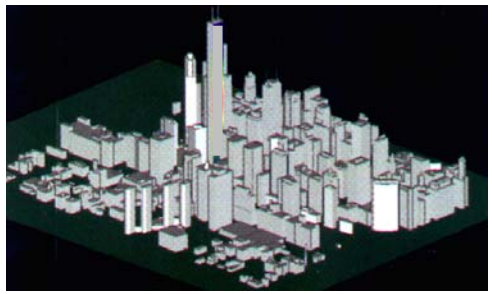
…Your new PC can run a $1000 \times 1000 \times 1000$ node FDTD or MAFIA simulation in a snap. Fine. So what ? …

Complex vs. Complicated

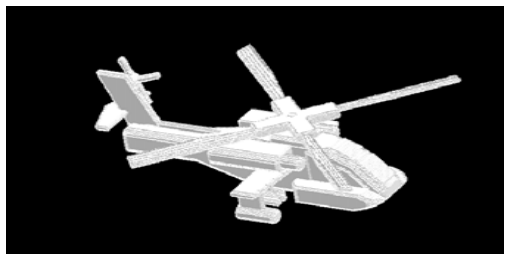
Many (in fact ***most***) EM boundary value problems of practical interest are ***both*** complex and complicated



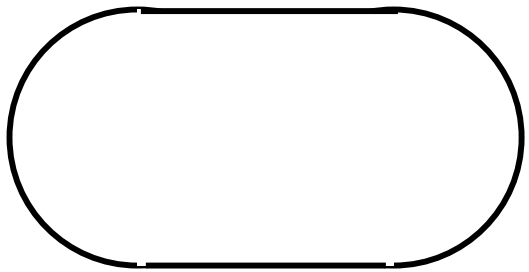
The CERN-LHC beam pipe



Urban area propagation



Apache-helicopter RCS



Stadium shaped PEC resonator

Structural complexity in the ***response*** shows up in systems with (deceptively) ***simple*** geometrical and constitutive properties.

Complexity

[R.R. Parwani, *Complexity*, web-book available on line at http://staff.science.nu.edu/~parwani/complexity]

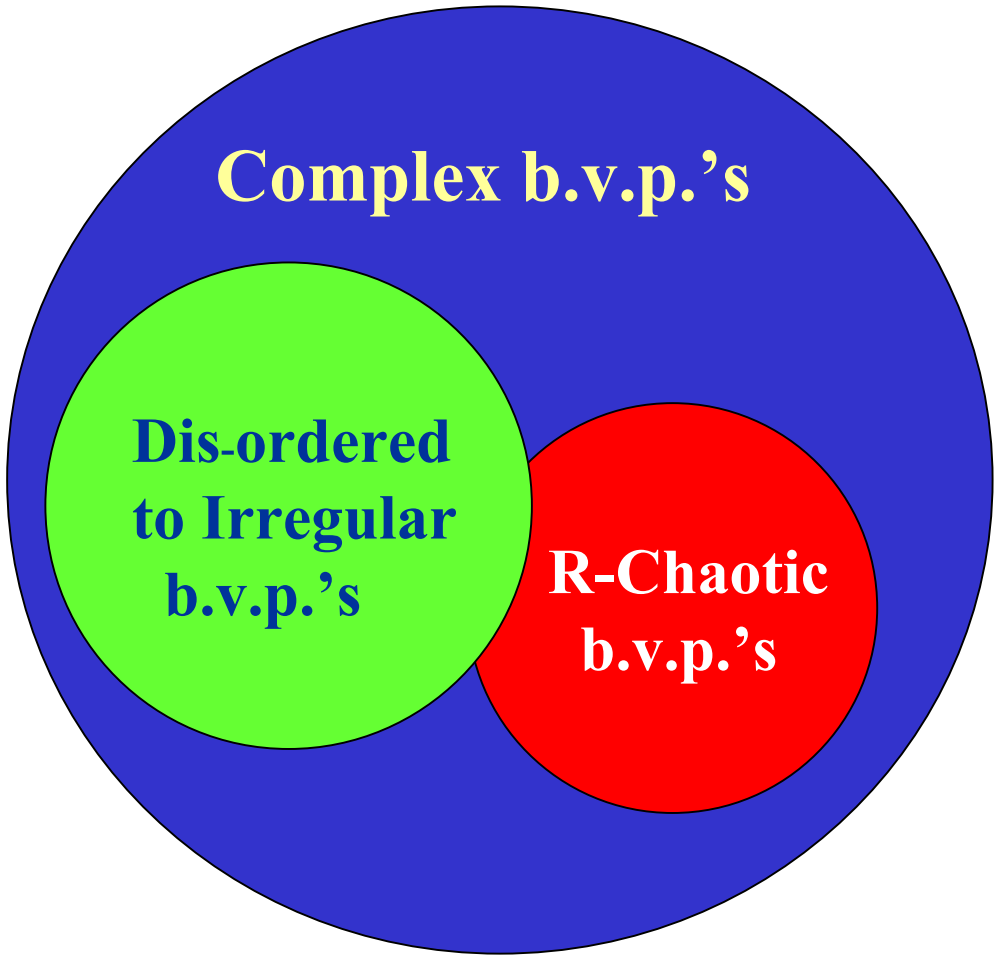
Defining ***complexity*** is…a ***complex*** task ***!***

Complexity means a butterfly flapping its wings in Jaipur causing a wind storm in Jamaica

We shall adopt the following tentative operational definition:

Knowledge (description) of a complex system is ***not e***quiv-alent to knowledge of its ***parts***. Mutual interaction is crucial. Reductionistic approaches do ***not*** work

Complex EM BVP's – a Venn Chart



STatistical ElectroMagnetics (STEM)

[T.H. Lehman, *A Statistical Theory of Electromagnetics in Complex Cavities*, IN-493, Phillips Labs., Kirtland, 1993]

In many practical cases, a ***statistical*** characterization of the EM field is appropriate and desirable, e.g. to ascertain

- whether the field at some critical spots might ever exceed some critical level (shielding, EMI/EMC problems),
- whether the average field within a given area will reach a specified level (broadcast coverage problems).

It turns out that a for many EM b.v.p.'s involving ***complex environments***, a statistical description is the only ***meaningful*** one (if not the ***only possible***).

Ergodicity : The Random Plane Wave Model

Under extreme multiple-scattering assumptions, the field at any point in a complex/irregular scenario can be thought of as a superposition of a *large* number *N* of plane waves with *uniformly distributed* phases, directions of arrival and polarizations (RPW ansatz).

Following an ergodic assumption, the field (spatial component) values might be expected to be *gaussian-distributed*. This turns to be the case even for $N \sim 10^2$ waves, *almost irrespective* of the detailed distribution of the amplitudes.

[A. Abdi, H. Hashemi, and S. Nader-Esfahani, "*On the PDF of a Sum of Random Vectors*", IEEE Trans. Comm., **48**, 7, 2000]

Random Plane Waves, cont.d - Correlations

Under the RPW assumption, the (locally smoothed, tensor) spatial field correlations take the form [B. Eckhardt, U. Dorr, U. Kuhl and H.-J. Stockmann, "*Correlations of Electromagnetic Fields in Chaotic Cavities*", Europhys. Lett. **46**, 134, 1999]:

$$\left\langle \vec{E}(\vec{x}+\vec{r}/2)\vec{E}(\vec{x}-\vec{r}/2)\right\rangle / \left\langle |\vec{E}|^2 \right\rangle = \begin{pmatrix} f_\perp & 0 & 0 \\ 0 & f_\perp & 0 \\ 0 & 0 & f_\ell \end{pmatrix}$$

$$f_\perp = \frac{3}{2}\left(\frac{\sin\xi}{\xi} - \frac{\sin\xi - \xi\cos\xi}{\xi^3}\right), \quad f_\ell = 3\left(\frac{\sin\xi - \xi\cos\xi}{\xi^3}\right), \quad \xi = kr$$

viz., loosely speaking, the field correlation distance is ~λ.

Random Plane Waves, cont.d – Degrees of Freedom

- **At any fixed resolution level (set by roundoff and/or measurement uncertainty) the field in a (compact) set *D* can be reconstructed (e.g. by cardinal interpolation) from a *finite number* ~ $\beta^n \mu_n(D)$ of samples taken on a regular grid with spacing ~ λ.**
  [O.M. Bucci and G. Franceschetti, "*On the Degrees of Freedom of Scattered Fields*", IEEE Trans. **AP-37**, 918, 1989]

- **Under the RPW assumption, the above field samples will be *gaussian distributed* , *almost independent,* and *bounded* (up to any prescribed confidence level), so that at any fixed resolution level (…) the needed field (component) samples may take only *a finite number* of *distinct values*.**

Random Plane Waves, cont.d - Coding

- **In the RPW assumption, all possible (distinct) fields in a compact set *D* correspond to all *finite-length words* constructed using a *finite alphabet*.**

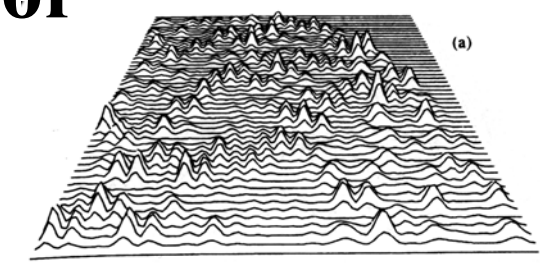
- **This allows to define a Shannon entropy, and to compute the *probability of occurrence* of *any* possible (distinct) field pattern in *D*.**
[P.Corona, D. Cuomo, I.M. Pinto, "*An Information Theoretic Description of EM Fields in Complex Environments*", IEEE Trans. **IM-36**, 1020, 1987]

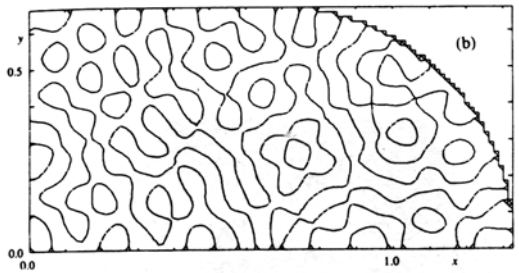
Random Plane Waves, cont.d - Credits

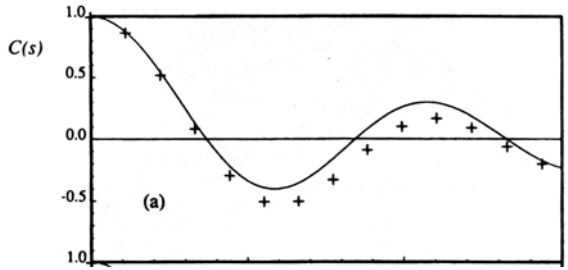
The random-plane-wave ansatz was formulated by Sir M.V. Berry in connection with (internal) ray-chaotic b.v.p.'s, and is named after him [M.V. Berry, "*Regular and Irregular Semiclassical Wave functions*", J. Phys. **A10**, 2083, 1977]

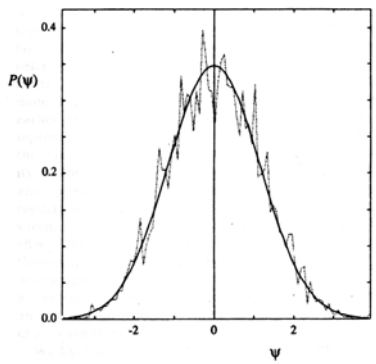
The RPW ansatz has been independently used by D. Hill, in his study of reverberation enclosures [D. Hill, Trans. IEEE **EMC-40**, 209, 1998; ibid., **EMC-44**, 95, 2002]

First Numerical Evidence in Favor of Berry's RPW Ansatz [1989]









PHYSICAL REVIEW A VOLUME 37, NUMBER 8 APRIL 15, 1988

Wave chaos in the stadium: Statistical properties of short-wave solutions of the Helmholtz equation

Steven W. McDonald* and Allan N. Kaufman
Physics Department and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 13 July 1987)

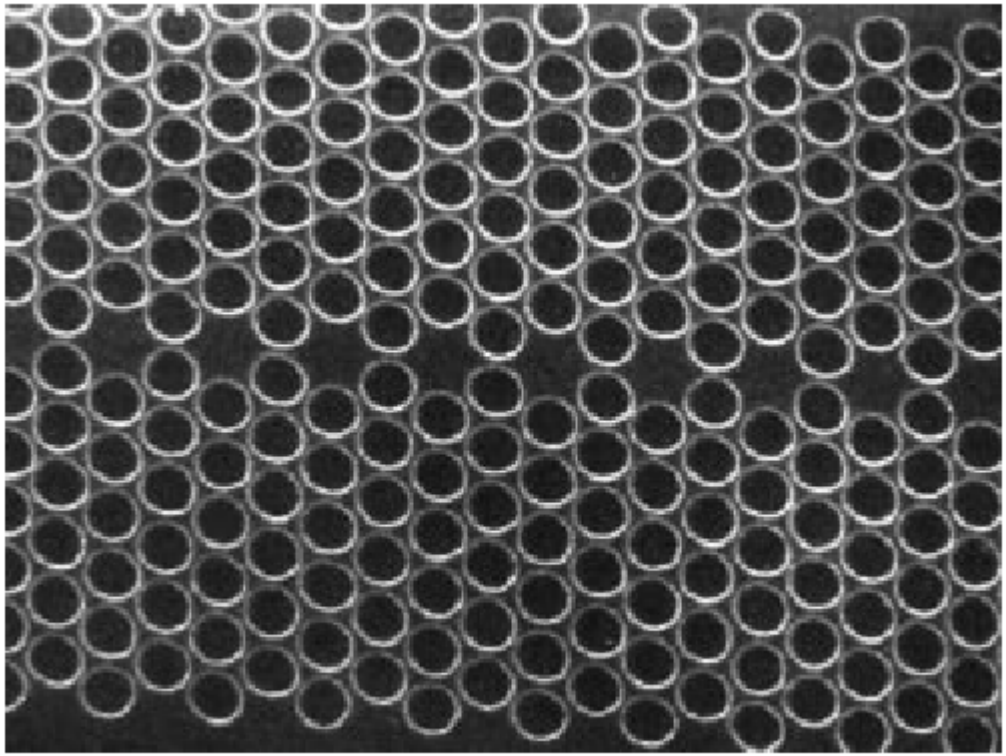
We numerically investigate statistical properties of short-wavelength normal modes and the spectrum for the Helmholtz equation in a two-dimensional stadium-shaped region. As the geometrical optics rays within this boundary (*billiards*) are nonintegrable, this wave problem serves as a simple model for the study of quantum chaos. The local spatial correlation function $\langle \psi_n(x+\frac{1}{2}s)\psi_n(x-\frac{1}{2}s)\rangle$ and the probability distribution $P_n(\psi)$ of wave amplitude for normal modes ψ_n are computed and compared with predictions based on semiclassical arguments applied to this nonintegrable Hamiltonian. The spectrum is analyzed in terms of the probability $P(\Delta E)$ of neighboring energy-eigenvalue separations, which is shown to be similar to a Wigner distribution for the eigenvalues of a random matrix.

Localization

In 1958 P.W. Anderson calculated the effects of perturbations (dis-order) on a 2D periodic lattice on the eigenvalues/eigenfunctions of Schrödinger equation. He showed that for a variety of dis-orders, the eigenfunctions become ***exponentially localized*** in the lattice (Floquet picture destroyed), as an effect of the interference from oddities-scattered waves.

[P.W. Anderson, "*Absence of Diffusion in Certain Random Lattices*", Phys. Rev. **109**, 1492, 1958]

Localization: PBG waveguides



(b)

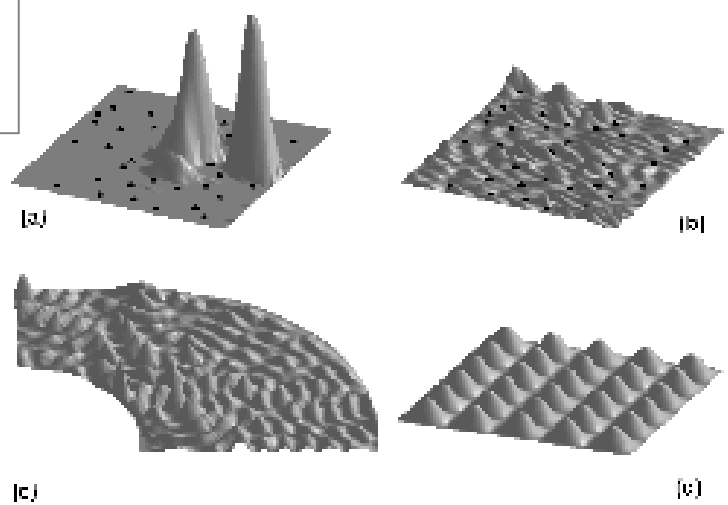
[A. Scherer, O. Painter, J. Vuckovic, M. Lonkar, T. Yoshie, "Photonic Crystals for Confining, Guiding and Emitting Light", IEEE Trans. **NT-1**, 4, 2002.]

Localization in Complex-Irregular Environments

[P. Pradhan and S. Sridhar, "*Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities*", Phys. Rev. Lett., **85**, 2360, 2000]

irregular-localized
(f =3.04 GHz)

irregular-ergodic
(f =7.33 GHz)



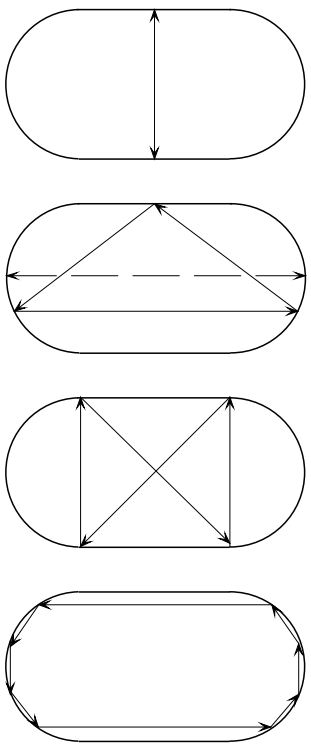
chaotic (ergodic)

separable

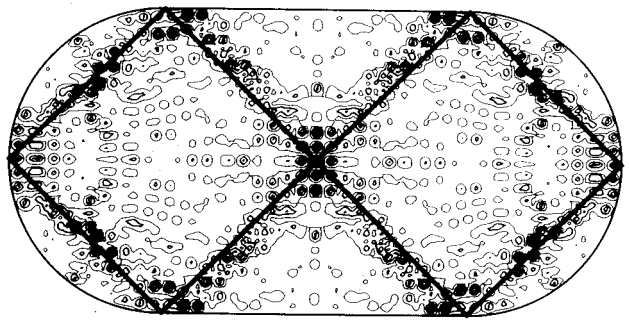
(a,b,d) 44cm × 21.8 cm square pillbox resonator.
(a,b) 36 random-uniformly placed Ø 1cm PEC circular tiles.
(c) Sinai quarter billiard.

Localization in Complex-Chaotic Environments: Scarred Eigenstates (in b.v.p.'s w. *mixed* ray kinematics)

A gallery of unstable (or marginally-stable) closed stadium orbits



Scar = anomalous eigenfunction intensity concentration at the location of isolated unstable ray-orbits.



First predicted by Heller [Phys. Rev. Lett., **53**,1515, 1984]; first observed by Sridhar [Phys. Rev. Lett., **67**, 785, 1991] in a microwave pillbox resonator experiment.

Scar statistics:

[T.M. Ansonsen, E. Ott, Q. Chen and R.N. Oerter, "*Statistics of Wavefunction Scars*", Phys. Rev. **E51**, 111, 1995]

Why studying R-chaotic EM BVP's

1. Because they provide *a viable* (!) *route* to understand the properties of generic (and otherwise untractable) ***complex / irregular*** b.v.p.'s - at least under suitable circumstances (extreme diffusion limit);

2. Because they pose a ***fundamental issue of principle*** about the hallmarks (if any) of ray - chaos in the short but full-wave regime, where the diffraction limit forbids the existence of structure at all scales below λ;

3. Because some of their ***unique*** *properties* might open the door to the design of *new devices* and *new artificially-engineered materials*.

2.

Chaos hallmarks (προτοφανεια) in the (short but full) wave regime

Quantum (Wave) Chaology

•Helmholtz equation

$$\nabla^2 E + k^2(z)E = 0$$

is the same as Schrödinger equation (SE) for point particle with mass m and energy U in a potential:

$$k^2(z) = \frac{2m[U - V(z)]}{\hbar^2}$$

•Short wavelength ($\lambda \to 0$) stationary EM fields ruled by Helmholtz equation correspond to semiclassical ($\hbar \to 0$) stationary solutions of Schrödinger equation.

•Most of the relevant papers are written in a quantum-mechanical perspective – although the main results do apply to *all sorts* of wave phenomena.

Digression I - Historical

The statistical nature of (quantum-mechanical) wave-scattering by (complex) many-body systems first shew up in nuclear ballistics .
[T. Ericson, Phys. Rev **5**, 430, 1960]

It was an early intuition by Ericson and Mayer that "...*nuclear reaction theory is equivalent to the theory of waveguides...*", in this respect .
[T. Ericson and T. Mayer-Kuckuck, Ann. Rev. Nucl. Sci., **16**, 183, 1966]

Similar features are observed in ballistic microstructures, where a single-electron wavefunction remains coherent across the whole system (meso-scopic systems, quantum-dots) .
[P.A. Mello, Pramana J. Phys., **56**, 425, 2001]

Quantum (Wave) Chaology explored using Microwaves

ELSEVIER

Physica E 9 (2001) 571–577

PHYSICA E

www.elsevier.nl/locate/physe

Microwave studies of chaotic billiards and disordered systems

Hans-Jürgen Stöckmann *, Michael Barth, Ulrich Dörr, Ulrich Kuhl, Hendrik Schanze

Fachbereich Physik, Philipps-Universität, Renthof 5, D-35032 Marburg, Germany

Abstract

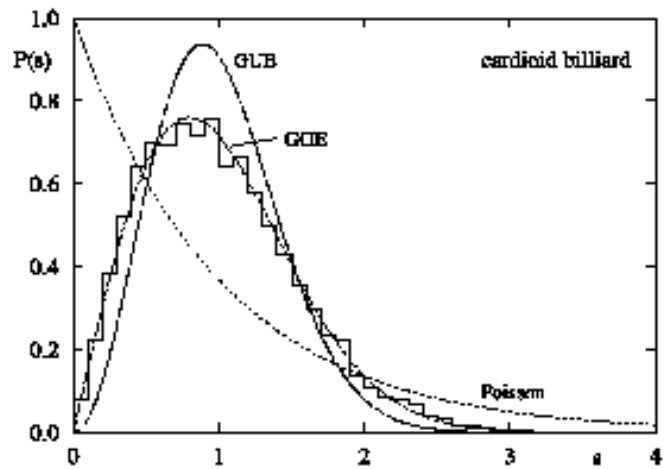
A review is given on our recent microwave results in chaotic systems with special emphasis on examples having a direct relevance for real mesoscopic systems. We present tests of predictions by Simons and Altshuler on velocity distributions of different types of spectral level dynamics, studies of transmission fluctuations through open billiards, and of the localization–delocalization transition of wave functions in disordered systems. The review ends with a presentation of chaotic field distributions in three-dimensional billiards. © 2001 Elsevier Science B.V. All rights reserved.

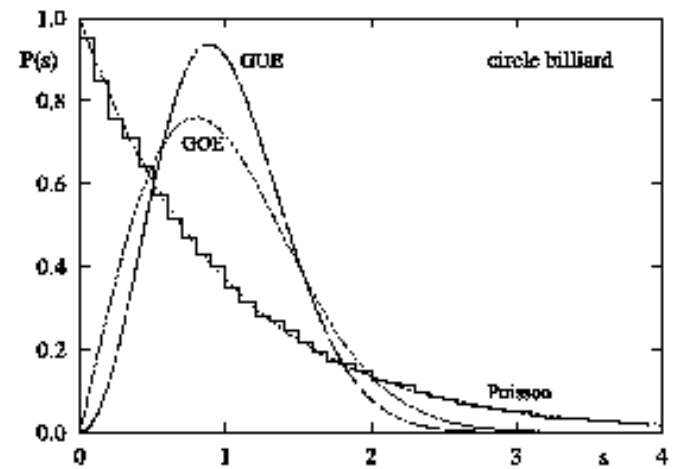
PACS: 05.45.Mt; 42.25.Dd; 73.23.–b

Keywords: Chaotic billiards; Velocity correlations; Conductance fluctuations; Wave functions in disordered systems

R-Chaos Hallmarks in the (short) wave regime:
Asymptotic Distributions of Eigenvalue Spacings

Normalized spacings: $s_n = \left(k_{n+1}^2 - k_n^2\right) / \left\langle \Delta k^2 \right\rangle$

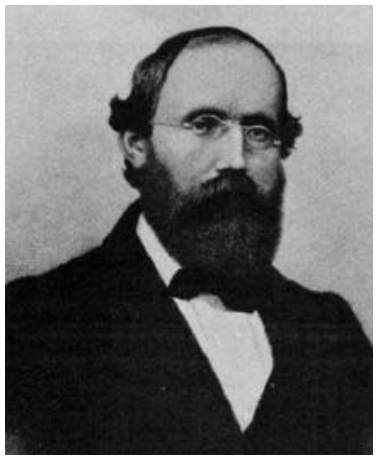


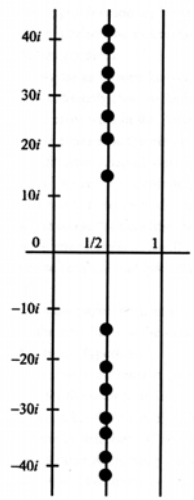


RMT
- **Poisson = integrable (no ray chaos);**
- **GOE=R-chaotic, time reversible;**
- **GUE=R-chaotic, time irreversible;**
- **(nonreciprocal, e.g. ferrite, inset);**

[M.L. Mehta, *Random Matrices*, Academic, NY, 1967]

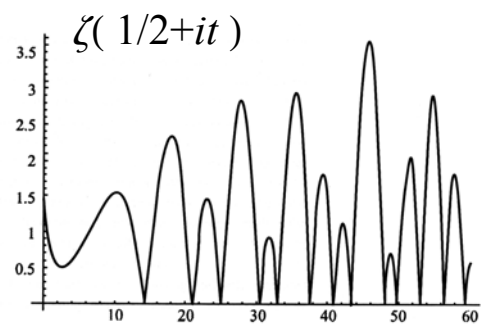
Digression II -Wave Chaology meets Number Theory: The Zeta Function Puzzle

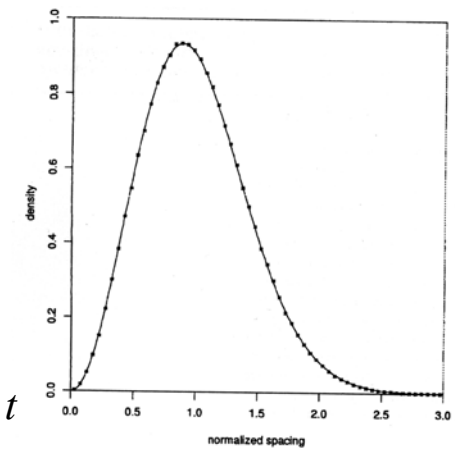




Bernhard Riemann (1826-1866)

Riemann hypothesis (1859): the non-trivial zeros of ζ(s) lie on $s = 1/2+it$, $t \in \mathcal{R}$ has *never* been disproved, yet neither proved.





$$\zeta(s) = \prod_{p \equiv primes} (1 - p^{-s})^{-1} =$$

$$= f(s) \prod_{z \equiv \zeta\ s\, zeros} (1 - s/z)$$

High order ζ(s) zeros fit GUE statistics. If the zeros of ζ(s) were the eigenvalues of some b.v.p., Riemann hypothesis would be proved.[Polya].

Why studying R-chaotic EM BVP's

1. Because they provide *a viable* (!) *route* to understand the properties of generic (and otherwise untractable) ***complex / irregular*** b.v.p.'s - at least under suitable circumstances (extreme diffusion limit);

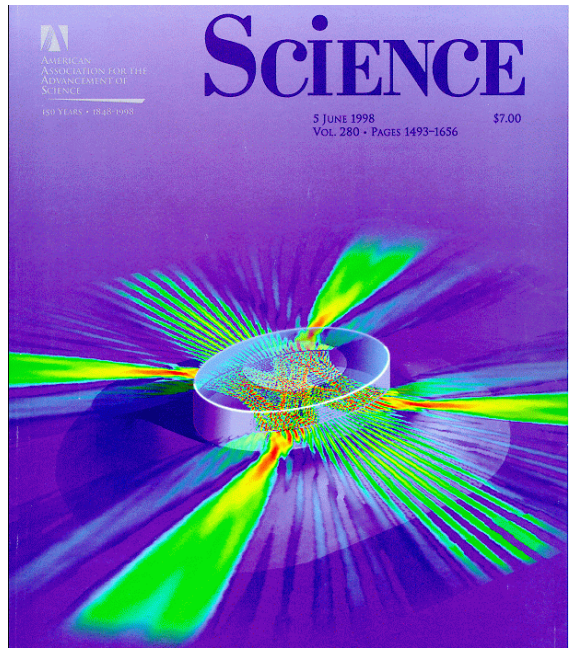
2. Because they pose a ***fundamental issue of principle*** about the hallmarks (if any) of ray - chaos in the short but full-wave regime, where the diffraction limit forbids the existence of structure at all scales below λ;

3. Because some of their ***unique*** *properties* might open the door to the design of *new devices* and *new artificially-engineered materials*.

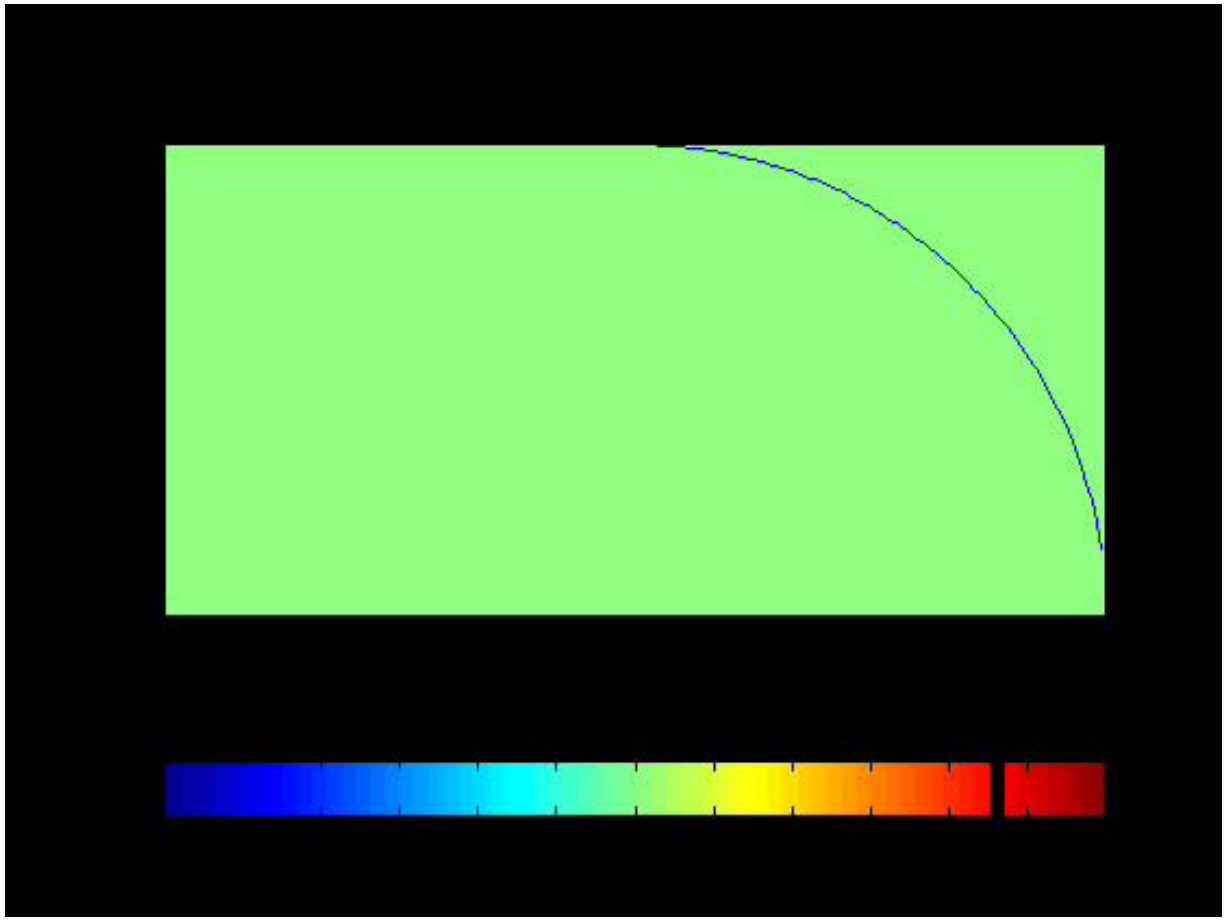
3.

New devices and (artificially engineered) materials from R-chaotic b.v.p.'s

… The Bow-tie Solid-State Lasers…

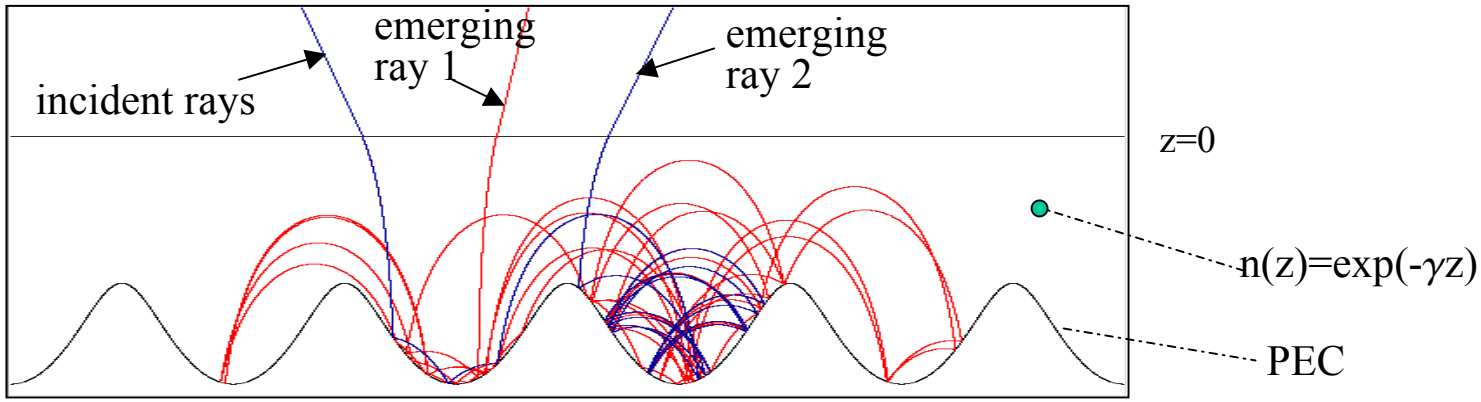


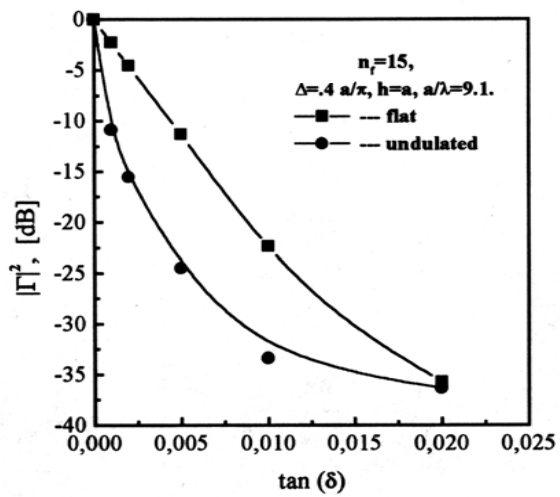
…Pulsed Reverberation Enclosures: a Movie…

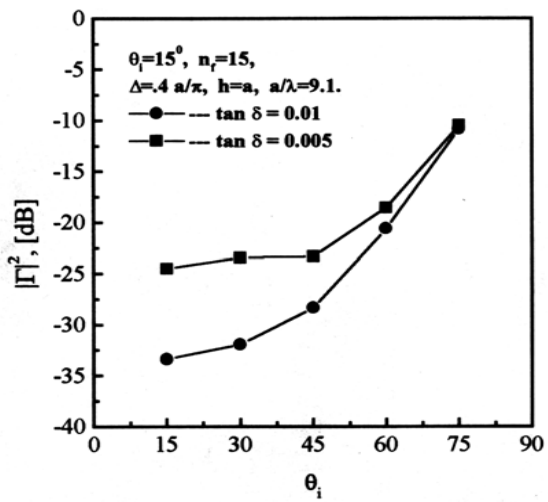


[V. Fiumara, V. Galdi, V. Pierro, I.M. Pinto, Proc. ICEAA 1999, invited paper]

…Smart Absorbers & Radar countermeasures (next talk)…







Italians: They do Speak Chaos !

PHYSICAL REVIEW E VOLUME 61, NUMBER 4 AP

Microwave study of quantum *n*-disk scattering

Wentao Lu, Lorenza Viola, Kristi Pance, Michael Rose, and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 16 September 1999)

We describe a wave-mechanical implementation of classically chaotic *n*-disk scattering based on thin two-dimensional microwave cavities. Two-, three-, and four-disk scatterings are investigated in detail. The experiments, which are able to probe the stationary Green's function of the system, yield both frequencies and widths of the low-lying quantum resonances. The observed spectra are found to be in good agreement with calculations based on semiclassical periodic orbit theory. Wave-vector autocorrelation functions are analyzed for various scattering geometries, the small wave-vector behavior allowing one to extract the escape rate from the quantum repellet. Quantitative agreement is found with the value predicted from classical scattering theory. For intermediate energies, nonuniversal oscillations are detected in the autocorrelation function, reflecting the presence of periodic orbits.

PACS number(s): 05.45.Mt, 05.45.Ac, 03.65.Sq, 84.40.−x

Chaotic Scattering of Microwaves

Tsampikos Kottos†, Uzy Smilansky†, Joaquim Fortuny‡ and Giuseppe Nesti‡

† Department of Physics of Complex Systems,
The Weizmann Institute of Science, Rehovot 76100, Israel

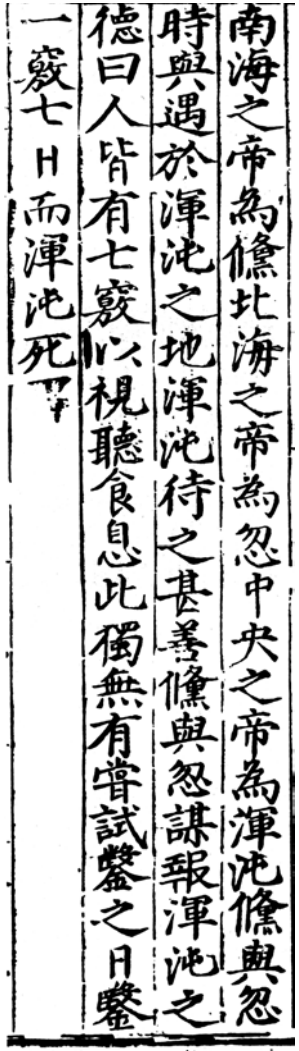
‡ Space Applications Institute and Joint Research Institute
I-21020 Ispra (VA), Italy

Radio Science, 34, 747, 1999

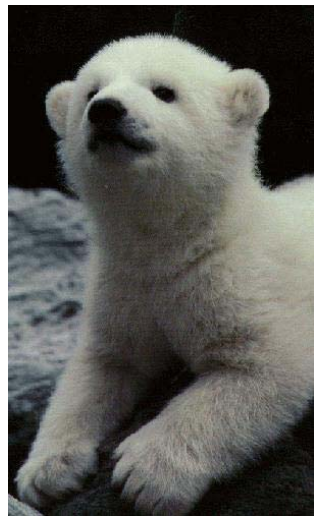
Abstract

In this paper we analyze a recent experiment conducted in an anechoic chamber, where the scattering of microwaves from an array of metallic cylinders was measured. This is a system which displays chaotic scattering in the short wave limit. The analysis of the experimental data is aimed at elucidating the effects of the underlying chaos. We describe a robust numerical method which provides the scattering matrix for any number of non overlapping reflecting cylinders. We use this method to calculate the scattering from one, two and three cylinders, and to compare the results in the numerical simulations with those obtained in the experiment. Both simulations and measurements validate the presented theory.

… Speaking of Chaos…


南海之帝為儵北海之帝為忽中央之帝為渾沌儵與忽
時與遇於渾沌之地渾沌待之甚善儵與忽謀報渾沌之
德曰人皆有七竅以視聽食息此獨無有嘗試鑿之日鑿
一竅七日而渾沌死

Chuang Tsu's *Tale of Chaos* (1525)
Ch'ing Dinasty roll.



Michael Angel, Author's baby, most relevant 2002's result out of his private chaos.