QUASI-RAY GAUSSIAN BEAM ALGORITHMS FOR FREQUENCY AND TIME DOMAIN PROPAGATION AND SCATTERING

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ABSTRACT

This paper contains a compact review of the basic theory and current applications of Gabor-based narrow-waisted quasi-ray Gaussian beam algorithms to frequency and time domain wave propagation and scattering in complex environments. Examples of application include aperture radiation, transmission through layered media, and rough surface scattering. Numerical results are presented for time-harmonic scattering of aperture-generated wavefields from moderately rough dielectric interfaces.

INTRODUCTION

Gaussian beams (GBs) have been used traditionally as well-collimated basis elements in a variety of frequency (FD) and time domain (TD) high-frequency/short-pulse radiation, propagation and scattering scenarios. In this connection, Gabor expansions in the (configuration)-(spectrum) FD or TD phase space provide a self-consistent way to decompose extended sources into GB basis functions which can be tracked individually through complex environments and eventually recombined to synthesize the wavefield at the observer [1-4].

A *nonconventional* form of the Gabor-based GB algorithm, which utilizes *narrow-waisted* (NW) poorly collimated GBs has been shown to yield particularly efficient and robust predictions for scattering by, and transmission through, complex environments irradiated by extended aperture distributions. In particular, it has been applied successfully to FD transmission through arbitrarily shaped dielectric layers [5-7] in connection with antenna radome design. More recently, it has been extended to FD and TD scattering from, and transmission through, moderately rough dielectric interfaces [8-10], and TD transmission through planar layers [11]. In the FD, the NW-GB basis functions can be tracked effectively via the *complex source point* (CSP) technique, which reduces the computationally intensive *complex* ray tracing for collimated GB propagation and scattering to *quasi-real* ray tracing, without the failure of strictly real ray field algorithms in caustic and other transition regions.

In this paper, we review some current applications of Gabor-based NW *quasi-ray* GB algorithms in propagation and scattering problems, involving radiation from extended apertures, propagation through layered media, and scattering from moderately rough surfaces. Both time-harmonic and pulsed excitations are considered, with the TD pulsed beam expansions obtained via analytic inversion from the FD [3, 4, 9-11].

GABOR-BASED NARROW-WAISTED GAUSSIAN BEAM ALGORITHMS: BASICS

To illustrate the basic theory underlying Gabor-based NW-GB algorithms, we consider a y-directed aperture field with assigned spatial distribution $E(x, z_A) = f(x)$ at $z = z_A$ in the two-dimensional (2-D) coordinate space (x,z). The aperture field distribution is to be parameterized in terms of a Gabor series,

$$f(x) = \sum_{m,n=-\infty}^{\infty} A_{mn} w(x - mL_x) \exp(in\beta_x x), \tag{1}$$

where A_{mn} are expansion coefficients, and $w(x)$ is a normalized Gaussian window. The representation in (1) places the aperture distribution on a discretized Gabor lattice (Fig. 1) in the (x,k_x) phase space indexed by (x_m,k_{xn}), with $(m,n) = 0,\pm 1,\pm 2,\ldots$, where k_x is the x-domain spectral wavenumber; L_x and β_x denote the spatial and spectral lattice periods, respectively, subject to the constraint $L_x\beta_x = 2\pi$. The linearly-phased Gaussian initial distributions in (1) give rise to GBs launched from the aperture plane, with the index m tagging the spatial displacement $x_m = mL_x$, and n tagging the spectral displacement (tilt), $k_{xn} = n\beta_x$ of the Gabor-GBs. The y-directed field radiated by the aperture into the halfspace $z < z_A$ can thus be written as

$$E(x,z) = \sum_{m,n=-\infty}^{\infty} A_{mn} B_{mn}(x,z), \tag{2}$$

with B_{mn} denoting the GB propagators. For *narrow-waisted* (NW) GBs with $L_x << d$ (d is the aperture width), the phase shift $\exp(in\beta_x x)$ produces spectra with $\beta_x = 2\pi/L_x >> 1$; in view of the GB spectral propagator $\exp\left[i\left(k_0^2 - k_{xn}^2\right)^{1/2}\right]$, where k_0 is the free space wavenumber, this renders all $n \neq 0$ GBs evanescent (i.e, non-contributing) sufficiently far from the $z = z_A$ plane. This property allows the computationally intensive determination of the excitation amplitudes A_{mn} in (1) to be avoided and replaced by sampling the aperture profile at the x_m lattice point locations. Field synthesis at the observer is accomplished by adding contributions from the ray-like $n = 0$ GBs. In the presence of (possibly stratified) inhomogeneities, the *complex ray* machinery, which allows *rigorous* tracking of the individual basis GBs through the environment, is shown to be simplified substantially for NW-GBs because this makes possible the use of efficient *quasi-real* ray tracing implemented through paraxial beam shooting [5-7]. Basically, the single multi-hop GB is tracked along its real-ray axis via conventional real ray tracing; the slightly complex NW-GB spectrum of the emerging GB is accounted for approximately through augmentation of its on-axis (real ray) value by a complex phase correction in the (perpendicular) distance from the axis to the off-axis observer [5-7].

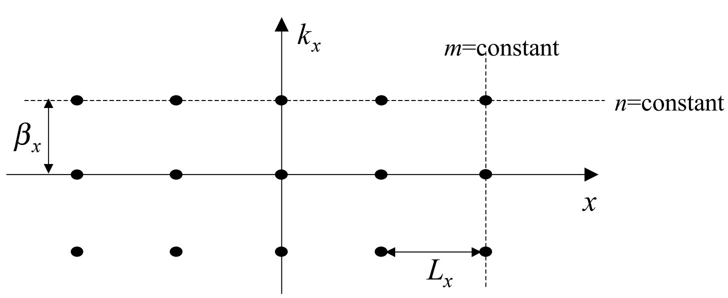


FIGURE 1 – Gabor-discretized phase space lattice.

LISTING OF PREVIOUS AND CURRENT APPLICATIONS

Aperture Radiation - Systematic investigation of both rigorous and approximate (paraxially asymptotic) Gabor-based GB parameterizations of time-harmonic wavefields radiated by large truncated plane aperture field distributions has been carried out in [1] (1-D apertures, 2-D fields) and [2] (2-D apertures, 3-D fields). The asymptotic NW-GB FD syntheses in [1, 2] have recently been extended to *pulsed* excitations [3, 4] leading to closed-form pulsed beam (PB) representations in terms of rapidly-computable analytic functions.

Propagation through Layered Media - The aperture-excited time-harmonic Gabor-based NW-GBs have been propagated through planar and curved dielectric layers by Maciel and Felsen [5, 6], using a quasi-real ray tracing (beam shooting) procedure to track the individual basis beams through the environment. These algorithms, originally restricted to 2-D geometries and time-harmonic excitation, have recently been extended to full 3-D (vector) configurations [7], and also to pulsed excitation for the simplest 2-D case of a single planar layer [11]. Applications to scattering from metallic objects coated by complex (possibly bi-anisotropic) multi-layered materials are currently under consideration.

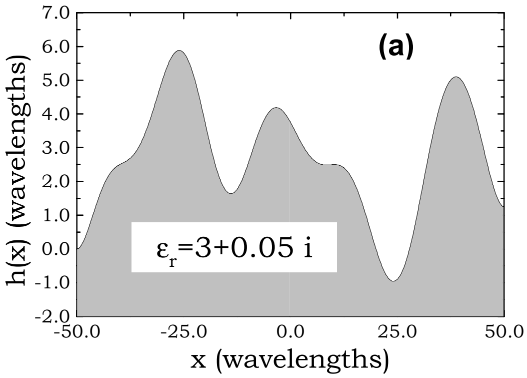
Rough Surface Scattering - Extension of the Gabor-based NW GB algorithms in [5, 6] to scattering from, and transmission through, moderately rough 1-D dielectric interfaces has been explored in [8] for time-harmonic excitation and, subsequently, in [9] for pulsed excitation. Applications to 2-D surfaces (3-D fields) are in progress [10]. Capabilities and limitations of these algorithms are discussed in [8-10]

As an example of application, we consider the lossy interface profile $h(x)$ in Fig. 2a, excited by a time-harmonic aperture field distribution with cosine tapering of width $d = 80\lambda_0$, located at $z_A = 6.4\lambda_0$ on the height scale of Fig. 2a, i.e., very close to the maximum profile height; λ_0 is the free-space wavelength. The reflected field observed at $z_{obs} = 20\lambda_0$ is shown in Fig. 2b. Here, $L_x = d/80 = \lambda_0$. On the plot, the beam-generated solution (dashed) agrees very well with the reference solution (solid) in this typical example. The same kind of accuracy is observed in the transmitted field beam GB synthesis.

Calibration and Computational Features – For all above algorithms, the range of applicability has been calibrated against independently generated rigorous numerical reference solutions. Whenever possible, accuracy assessments have also been formalized analytically in terms of critical nondimensional estimators. From the computational viewpoint, when dealing with electrically large domains, the NW-GB scheme tend to preserve the favorable features of standard ray-optical techniques, *without* failing in typical ray-field transition regions.

CONCLUSIONS

A compact review of current applications of Gabor-based NW *quasi-ray* GB algorithms for wave propagation and scattering in complex environments has been provided. Overall, it appears that for several problem categories these algorithms are able to furnish accurate predictions over calibrated parameter ranges with modest computational effort. Examples of application of these forward solvers in inverse scattering scenarios involving moderately rough interfaces are presented in a companion paper [12].



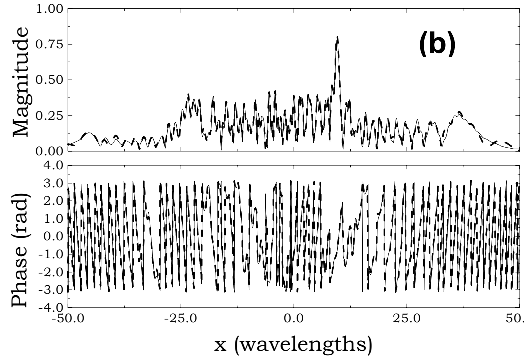


FIGURE 2 – (a): Rough surface geometry and parameters. (b): Beam-computed (dashed) and reference solutions (solid) for reflected field at $z_{obs} = 20\lambda_0$.

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