Modeling of Mode Excitation from Sources in Two-Dimensional PBG Waveguides

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Abstract. *An efficient algorithm for the evaluation of the Green's function and mode excitation by line sources in a two-dimensional PBG waveguide that is based on a method of moments formulation is presented.*

I. Introduction

The desire to achieve low-loss propagation in the millimeter-wave and optical ranges has recently motivated research into new ways of guiding electromagnetic waves. A photonic bandgap (PBG) material with a row of defects (missing elements) constitutes a waveguiding structure that provides an attractive alternative to conventional waveguides [1], [2]. The applicability of such materials for constructing devices such as switches, multi/demultiplexers, power dividers, couplers, etc., is also receiving increasing interest.

In this summary we demonstrate an efficient analytical/numerical method to model mode excitation and discontinuities inside a 2D PBG waveguide. The PBG waveguide consists of a periodic array of either metallic or dielectric posts, or holes in a dielectric slab, periodically spaced in the *x*-*z* plane. (For simplicity, the case of metallic posts is considered here). The structure is terminated at $y = 0$ and $y = h$ by metallic planes (generalization to other structures is possible). A row of missing posts in the *z* direction forms the waveguide channel (with propagation in the *z* direction). For generality, the PBG structure is allowed to be finite in the *x* direction, i.e., there is an arbitrary finite number of rows on either side of the channel. The method uses the one-dimensional (1D) periodicity properties of the waveguide along the *z* direction. To further improve computational efficiency, a 2D Ewald acceleration scheme [3], [4] is used to improve the convergence of the periodic free-space Green's function that is used in the unit-supercell analysis. This combination of techniques provides an efficient method to construct the Brillouin dispersion diagram of the PBG waveguide modes, and determine the fields of the modes.

The analysis technique is then extended to treat sources inside the PBG waveguide. A source is modeled as a line source with an impressed current. Similarly, a discontinuity in the form of a metallic post inside the channel can also be modeled. The near field from the source or discontinuity, as well as the amplitudes of the guided modes that are launched by the source, or transmitted and reflected by the discontinuity, can be directly determined. Numerical examples will be shown to demonstrate these calculations.

II. The Dispersion Diagram

The geometry of the PBG waveguide is shown in Fig. 1. It is a periodic structure along *z* with period *a*, and truncated along *x* with a finite number of rows on either side of the channel. A periodic "supercell" is defined by the dashed line. We assume that the electric field is polarized along the *y* axis, i.e., TE_z propagation (which is also TE_x in this 2D problem), since there is no variation along the *y* axis. We denote with $J(\mathbf{r})$ and $E(\mathbf{r})$ the surface current and electric field directed along *y*. The dispersion (Brillouin) diagram of the guided mode and the corresponding currents on the metallic posts are determined by solving the homogeneous version of the electric field integral equation (EFIE)

$$\int_{S_0} J_{\text{post}}^{\infty}(\mathbf{r}',k_z)\, G^{\infty}(\mathbf{r},\mathbf{r}',k_z)\, d\mathbf{r}' = E_{\text{post}}^{\infty}(\mathbf{r},k_z) = 0, \qquad \mathbf{r} = S_0, \tag{1}$$

which implies that the field E_{post}^{∞} produced by J_{post}^{∞} must vanish on the surface of the posts S_0. Note that in (1) the numerical integration is carried out on the supercell $n = 0$ only (see Fig. 1), since we use the periodic

Green's function $G^{\infty}\left(\mathbf{r},\mathbf{r}',k_z\right)$ for the field produced by a periodic array of line sources in free space successively phased as $\exp\left(-jk_z na\right)$. The EFIE (1) is then discretized according to the method of moments (MoM) procedure. The mutual coupling between the various basis-function elements (to construct the MoM matrix) is then evaluated efficiently using the periodic Green's function G^{∞} that is accelerated using the Ewald method modified for 2D geometries [3], [4]. Typically only two or three terms in the Ewald sums are sufficient to achieve good accuracy. The dispersion diagram is evaluated by finding the zeros of the determinant of the MoM matrix (yielding the wavenumber of the guided mode k_{z0}) for various radian frequencies ω. Once the modal current J^{∞}_{post} that satisfies (1) is found, the modal field E^{∞}_{post} can be determined. After the modal field is obtained, the amplitude of the guided mode can be established, although various definitions of modal amplitude are possible (e.g., the modal amplitude can be based on the total field at a particular reference point, or the field of only the 0th Floquet mode at a given reference point).

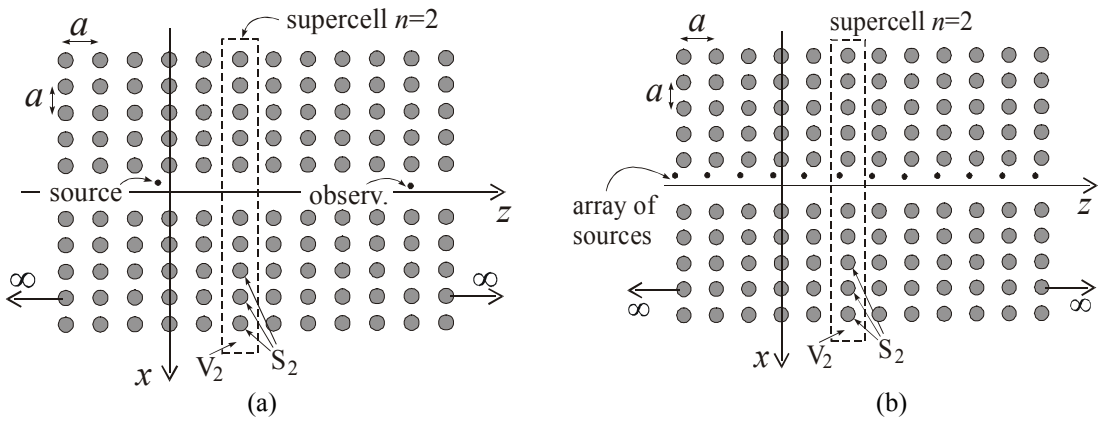

Fig. 1 (a) Geometry of the PBG waveguide and its excitation. The waveguide is an infinite periodic structure along z, with period a, and truncated along x. In the figure, the periodic supercell $n = 2$ is shown. The source is located in the $n = 0$ supercell. S_n denotes the surface of the conductors in the nth supercell. The volumetric region of the nth supercell is denoted by V_n. **(b)** An infinite array of sources located in the waveguide. The original source is the one in the $n = 0$ supercell. In the array scanning method, the infinite array is used to synthesize the field of the single impressed source. S_n denotes the surface of the conductors in the nth supercell.

III. Excitation of the PBG Waveguide

An electric line source $J_S(\mathbf{r}')$ is located in the $n = 0$ supercell at $\mathbf{r}' = \mathbf{r}_0 = (x_0, z_0)$, and the electric field is evaluated at an observation point $\mathbf{r} = (x, z)$ located in the waveguide. In the following, the radiation by a single source (and its modal excitation) in the waveguide is determined by using the "array scanning method" discussed below.

III-A The Array Scanning Method (ASM)

The first step is to define a relation between an infinite periodic array of line sources $J^{\infty}_S(\mathbf{r},k_z)$ with currents directed along y and a single line source $J_S(\mathbf{r}')$ as (J_S denotes volume current here)

$$J^{\infty}_S(\mathbf{r}',k_z)=\sum_{m=-\infty}^{\infty}\delta(z'-z_0-ma)\,\delta(x'-x_0)\,e^{-jk_z ma}, \qquad J_S(\mathbf{r}')=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}J^{\infty}_S(\mathbf{r}',k_z)\,dk_z, \tag{2}$$

in which k_z is an impressed wavenumber along z. The single line source $J_S(\mathbf{r}')$ is thus synthesized from the array of line sources by integrating in the phase-shift variable over the Brillouin zone. The electric field at any point $\mathbf{r}$, produced by the periodic array of line sources in free space is denoted as $G^{\infty}\left(\mathbf{r},\mathbf{r}_0,k_z\right)$. The field produced by the periodic array of line sources in the PBG waveguide environment is denoted by $G^{\infty}_{PBG}(\mathbf{r}, \mathbf{r}_0, k_z)$. By the same weighted superposition used in (3), the electric field produced by the single source $J_S(\mathbf{r}')$ in the waveguide is then given by

$$G_{PBG}(\mathbf{r},\mathbf{r}_0)=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} G_{PBG}^{\infty}(\mathbf{r},\mathbf{r}_0,k_z)\,dk_z\ . \tag{3}$$

The calculation of $G_{PBG}^{\infty}(\mathbf{r},\mathbf{r}_0,k_z)$ is discussed in the next section.

III-B The MoM Solution, Using the Array Scanning Method

In order to obtain $G_{PBG}^{\infty}(\mathbf{r},\mathbf{r}_0,k_z)$, the MoM is used to solve the EFIE in order to obtain the surface current $J_{\text{post}}^{\infty}(\mathbf{r},k_z)$ (periodic along z with period a) on the various metallic posts within the supercell, excited by the impressed periodic line-source array current $J_S^{\infty}(\mathbf{r}',k_z)$. The mutual coupling between the various basis-function elements (to construct the MoM matrix), is evaluated using the periodic free-space Green's function G^{∞} (which is accelerated using the Ewald method [3], [4]). The periodic electric field at any point $\mathbf{r}$, produced by the periodic current on the posts that make up the PBG structure, is evaluated by integrating the post currents over the single supercell S_0, as

$$E_{\text{post}}^{\infty}(\mathbf{r},k_z)=\int_{S_0} J_{\text{post}}^{\infty}(\mathbf{r}',k_z)\,G^{\infty}(\mathbf{r},\mathbf{r}',k_z)\,d\mathbf{r}'\ . \tag{4}$$

On the posts, this field has to cancel the field $G^{\infty}(\mathbf{r},\mathbf{r}',k_z)$ due to the periodic array of line sources in free space. This results in the EFIE that is solved numerically using the method of moments in order to obtain the post currents $J_{\text{post}}^{\infty}(\mathbf{r},k_z)$ within the supercell S_0. The periodic Green's function for the array of phased line sources in the PBG environment is then given as

$$G_{PBG}^{\infty}(\mathbf{r},\,\mathbf{r}_0,\,k_z)=G^{\infty}(\mathbf{r},\,\mathbf{r}_0,\,k_z)+E_{\text{post}}^{\infty}(\mathbf{r},\,k_z)\,. \tag{5}$$

III-C Mode Excitation

The current and the electric field in an arbitrary nth cell, induced by the *single* impressed line source in the cell $n = 0$, are evaluated using the ASM as

$$J_{\text{post}}(\mathbf{r}+na\,\hat{\mathbf{z}})=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} J_{\text{post}}^{\infty}(\mathbf{r},k_z)\ e^{-jk_z na}dk_z\,,$$

$$G_{PBG}(\mathbf{r}+na\,\hat{\mathbf{z}},\mathbf{r}_0)=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} G_{PBG}^{\infty}(\mathbf{r},\mathbf{r}_0,k_z)\ e^{-jk_z na}dk_z\,, \tag{6}$$

with $\mathbf{r}\in S_0$ for the post currents and $\mathbf{r}\in V_0$ for the G_{PBG} field. Assuming a mode with propagation constant $k_z = k_{z0}$ exists in the PBG waveguide, the terms $J_{\text{post}}^{\infty}(\mathrm{r},k_z)$ and $G_{PBG}^{\infty}(\mathbf{r},\mathbf{r}_0,k_z)$ have a pole at $k_z = \pm k_{z0}$. (The poles are periodically spaced in the k_z plane, and it is assumed here that k_{z0} is the pole that lies within the Brillouin zone of integration.) The residue evaluation at $k_z = k_{z0}$, or $k_z = -k_{z0}$ for $n > 0$ or $n < 0$, respectively, furnishes the modal current and electric field in the waveguide. Thus, the modal field is

$$G_{PBG}^{\text{mode}}(\mathbf{r}\pm|n|\,a\,\hat{\mathbf{z}},\mathbf{r}_0)=\mp ja\,R_G(\mathbf{r},\mathbf{r}_0,\pm k_{z0})\,\mathrm{e}^{-jk_{z0}|n|a}\ , \tag{7}$$

in which $R_G(\mathbf{r},\mathbf{r_0},k_{z0}) = \lim_{k_z\to k_{z0}}[(k_z-k_{z0})G_{PBG}^{\infty}(\mathbf{r},\mathbf{r_0},k_z)]$. Similar concepts apply to the post currents $J_{\text{post}}(\mathbf{r}+na\,\hat{\mathbf{z}})$.

IV. Numerical Results

As a simple example we analyze a waveguide as in Fig.1a, with metallic rods of radius $r = 0.2a$, where a is the period. The lateral rows of the waveguide, assuming an infinite number of rows on each side, has its first complete stop band (0th band gap) for the TEz polarization when $0< a/\lambda_0 <0.48$ (λ_0 is the free-space wavelength), while the 1*st* band gap is for $0.72< a/\lambda_0 <0.83$ [5]. Since we would have multimode

propagation in the waveguide for frequencies in the 1st band gap, we analyze here the dispersion diagram of the first mode in the 0th band gap represented below the dashed area in Fig. 2a. The waveguide is modeled as in Sec. II, assuming four rows on each side of the defect-channel. The Brillouin diagram for the mode phase constant β along the z-axis of the waveguide (see Fig.1a) shows a cutoff frequency for $a/\lambda_0 = 0.27$.
Next, we place an electric line current within the waveguide at (x, z)=(0, 0), at a normalized frequency $a/\lambda_0 = 0.3$. In the Table are reported the orders of magnitude of the induced current on each of the four rows on the sides of the channel. (There is some variation with n, but all the post currents in a given row have the same order of magnitude.) Note that even three lateral rows would suffice to propagate the signal along the waveguide since the current is decreased of 3-4 orders of magnitude with respect the first row. Finally, Fig. 2c plots the real part of the electric field E_y on the axis of the waveguide, sampled at locations $z_n = na$, the center of the nth supercell (see Fig.1). The field is evaluated using (5) with (4). It is clear that there is a propagating mode, with guided wavelength $\lambda_g = 8.0a$, which is verified by data from the Brillouin diagram in Fig. 2a, using the relation $\lambda_g = 2\pi / \beta$.

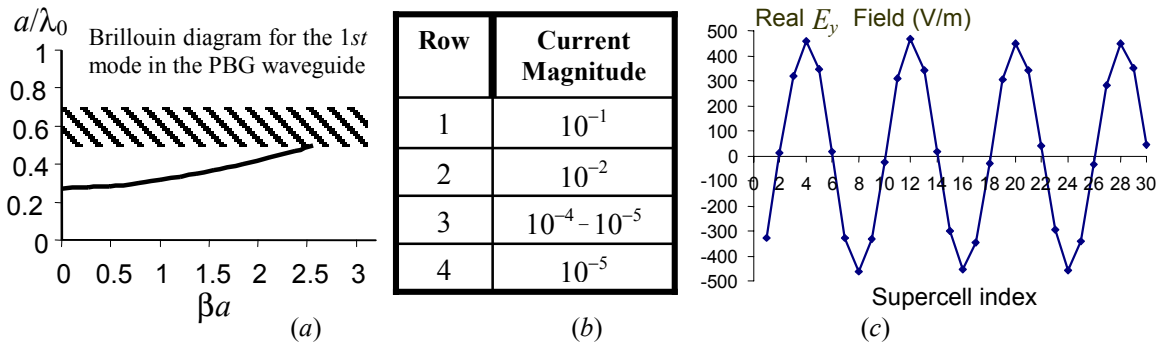

Fig. 2 (a) Brillouin diagram for the PBG waveguide in the 0th band gap (below the dashed area) of the surrounding PBG structure that consists of an array of metal posts. **(b)** Table representing the orders of magnitude of the induced current on each lateral row. **(c)** Real part of the electric field E_y sampled at the center of each supercell.

V. Conclusions

An efficient algorithm for the evaluation of the Green's function and modal excitation from a single source in a PBG waveguide has been derived using an array-scanning method. Once the Green's function is known, the same formalism (not discussed here) can be applied to the evaluation of the reflected and transmitted modes by a discontinuity in the PBG waveguide. In the present summary the algorithm has been specialized to a 2D PBG waveguide made of metallic posts in order to simplify the notation and the formulation, although the algorithm can also be applied to 2D and 3D PBG waveguides made of dielectric material. The numerical efficiency is improved by using the Ewald method to accelerate the periodic free-space Green's function, which also permits a fast determination of the Brillouin diagram.

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