EXCITATION OF SURFACE WAVES AND LEAKY WAVES IN FINITE PERIODIC ARRAYS OF PRINTED DIPOLES

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Abstract

Recently, a hybrid (high frequency)-(method of moments) technique has been proposed for the analysis of large periodic phased arrays. This technique is based on a Floquet wave diffraction theory for the representation of the radiation from and scattering by finite periodic structures. Through an appropriate 'fringe' integral equation the edge-induced current is interpreted as associated to wave diffraction at the boundaries of the array, thus allowing an efficient representation in terms of a few array-domain basis functions. In this paper, this technique is applied to the analysis of microstrip arrays. In particular focus will be given to the effects associated to the excitation of surface waves and leaky waves in the finite periodic array.

INTRODUCTION

Periodic arrays have been deeply investigated for a series of interesting electromagnetic properties. Practical applications of periodic structures in electromagnetic engineering are phased arrays, reflect-arrays, frequency selective surfaces (FSS), polarizers, photonic bandgap (PBG) materials. The estimation of array parameters is usually based on the hypothesis of infinite periodic structure [1]; this allows the expansion of the fields and currents in terms of Floquet waves (FW's), thus reducing the analysis to that of a single cell of periodicity. Nevertheless, the infinite array assumption cannot predict the current perturbation associated to the finite extent of the actual structure [2]. On the other hand, a rigorous analysis of the finite array based on an element-by-element method of moments (MoM) becomes computationally prohibitive as the size of the array increases. Recently, a method [3] has been proposed for the electromagnetic analysis of large periodic arrays. In this method, the infinite array Floquet-type current is augmented by an edge-induced current which may be rigorously estimated through the numerical solution of an appropriate 'fringe' integral equation. The fringe current is expanded in terms of a few array-domain basis functions, whose structure is derived from the high-frequency ray-diffraction representation of the field radiated by the array. The theoretical basis of the method is given in a series of recent papers (see [4] and references therein), which generalize to finite arrays the FW representation of infinite periodic arrays.

In [5], the method was applied to the analysis of arrays in layered media, by referring to a prototypical case of a finite periodic array of printed dipoles on a grounded dielectric slab. Here, with reference to the same structure, the phenomena associated with the excitation of guided waves is investigated with emphasis to the description of the perturbation which the periodic loading exerts on the of the guided wave propagation constants of the bare slab. A modification of the algorithm in [5] is adopted to properly describe the relevant phenomenology. Numerical results will show the effectiveness of the method.

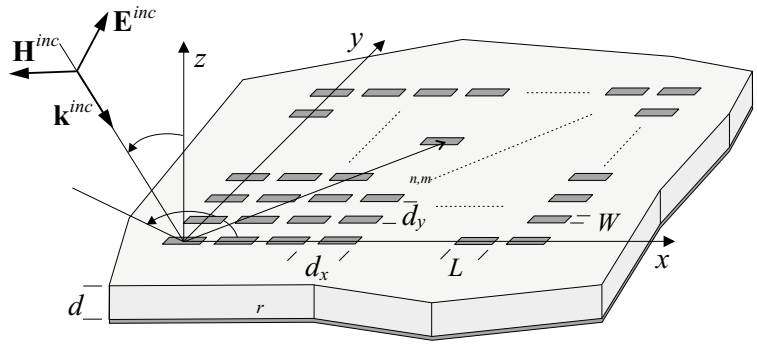

Fig. 1. Finite rectangular periodic array of conducting printed dipoles on an infinite dielectric grounded slab illuminated by an incident plane wave. The dipoles are oriented along x axis, with length denoted by L and width by W, and are arranged on a rectangular grid, with spatial periodicity denoted by d_x and d_y. The number of dipoles is N_x, N_y along x, y directions, respectively. The dielectric slab is characterized by thickness d and relative permittivity ε_r.

FORMULATION

The geometry investigated in this paper is a finite rectangular periodic array of printed dipoles, as shown and described in Fig. 1. The array is illuminated by an incident plane wave, with electric field $\mathbf{E}^{inc}(\mathbf{r}) = \mathbf{E}_o e^{-j\mathbf{k}^{inc}\cdot\mathbf{r}}$ where $\mathbf{r}$ is the position vector and $\mathbf{k}^{inc}$ is the wave vector of the incident plane wave. An $e^{j\omega t}$ time dependence is assumed and suppressed throughout the paper.

An unknown electric current distribution $\mathbf{J}$ is assumed on the surface of the printed dipoles. Imposing the vanishing of the electric field at the perfectly conducting dipoles yields the following electric field integral equation (EFIE)

$$\chi_A \mathbf{E}^s_{tan}(\mathbf{J}) = -\chi_A \mathbf{E}^{imp}_{tan}, \tag{1}$$

where $\mathbf{E}^s_{tan}(\mathbf{J})$ and $\mathbf{E}^{imp}_{tan}$ are the tangential components of the electric field scattered by the unknown current $\mathbf{J}$ and of the impressed electric field, respectively. In (1), A denotes the region occupied by the conducting dipoles and χ_A is the characteristic function of A [3].

As detailed in [3][5], the EFIE in (1) can be decomposed into two coupled integral equations (IE's)

$$\chi_\infty \mathbf{E}^s_{tan}(\mathbf{J}_\infty) = -\chi_\infty \mathbf{E}^{imp}_{tan} \tag{2a}$$
$$\chi_A \mathbf{E}^s_{tan}(\mathbf{J}_d) = \chi_A \mathbf{E}^f. \tag{2b}$$

The first IE is the one associated with the infinite periodic array problem. The second IE is the fringe integral equation (FIE), whose unknown is the fringe current $\mathbf{J}_d = \mathbf{J} - \chi_A \mathbf{J}_\infty$ and the forcing term $\mathbf{E}^f = \mathbf{E}^s_{tan}[(\chi_\infty - \chi_A)\mathbf{J}_\infty]$ represents the electric field scattered by the external periodic array, complementary to the actual one [3]. The current of the actual finite array is expressed as the sum of the PO current $\chi_A \mathbf{J}_\infty$ (the infinite array current windowed on the finite array region) and the fringe current $\mathbf{J}_d$.

INTEGRAL EQUATIONS SOLUTION

The solution procedure is developed in two subsequent steps. First, the infinite periodic array IE (2a) is solved using a conventional MoM approach [1]. Next, we solve the FIE (2b) through an hybrid asymptotic-MoM algorithm.

The current $\mathbf{J}_\infty$ of the infinite periodic array respects Floquet conditions. Consequently the solution of the infinite periodic array problem is reduced to that of a single periodic cell, with

a negligible numerical effort. By applying a Galerkin procedure, the IE (2a) is converted into a linear system $[Z]_\infty[I]_\infty = [V]_\infty$, where the matrix elements are in terms of a double infinite series of Floquet harmonics.

The second step is the numerical solution of the FIE (2b). First, we note that the problem associated to the FIE, is a finite periodic array with an aperiodic excitation. By applying the truncated FW diffraction theory [4], the forcing term can be asymptotically expressed in terms of two kind of contributions: 1) FW-induced diffracted waves and 2) FW-induced guided waves, excited at the edges and vertices of the array, i.e.,

$$\mathbf{E}^f \sim \sum_h (U_h^{d,e}\mathbf{E}_h^{d,e} + \mathbf{E}_h^{d,v}) + \sum_i (U_i^{GW,e}\mathbf{E}_i^{GW,e} + \mathbf{E}_i^{GW,v}) \tag{3}$$

where $\mathbf{E}_h^{d,e/v}$ denotes the FW-induced edge/vertex diffracted wave electric field, and $\mathbf{E}_i^{GW,e/v}$ represents the edge/vertex excited surface wave (SW) or leaky wave (LW) electric field. The truncation functions $U_{h/i}^{d/GW,e}$ bound the domain of the edge waves at the shadow boundaries arising from the end-points of each edge. The analytical expressions of the various terms involved in (3) can be found in [4]. In the expansion (3), LW's can be neglected, since their contributions rapidly decay away from the array border in this kind of configuration. The edge diffracted waves and SW's supported by the array are found by the generalized Fermat principle. The connection between the truncated FW's and the edge diffracted waves or SW's is established via phase-matching along the array edges, according to the following relationships

$$\mathbf{k}^{FW} \cdot \hat{\ell} = \mathbf{k}_h^{d,e} \cdot \hat{\ell} \qquad \mathbf{k}^{FW} \cdot \hat{\ell} = \mathbf{k}_i^{SW,e} \cdot \hat{\ell} \tag{4}$$

where $\mathbf{k}^{FW}$ is the FW wavevector, $\mathbf{k}_h^{d,e}$ and $\mathbf{k}_i^{SW,e}$ are the diffracted-wave and SW wavevector, respectively, and $\hat{\ell}$ is the unit vector along the array edge.

On the basis of the physical description gained by the high-frequency representation (3) of the FIE forcing term, the unknown fringe current $\mathbf{J}_d$ is expanded in terms of array-domain basis functions, associated to diffracted waves and to guided waves, as

$$\mathbf{J}_d \sim \sum_h a_h \mathbf{F}_h^d + \sum_i b_i \mathbf{F}_i^{GW} \tag{5}$$

where a_h and b_i are unknown coefficients to be determined. Each basis function include the edge contribution plus two end-point vertex contributions, which ensure the uniform continuity to the current. The edge diffracted-wave currents asymptotically propagates with the speed of light and decay like $1/\rho^{3/2}$ (where ρ is the distance from the edge), since are associated to slope-type space wave fields.

The edge-excited guided-wave currents decay as $1/\rho^0$. In order to determine the exact wavenumber of the guided-wave currents, the loading effect due to the presence of the periodic array of dipoles must be properly taken into account by solving the homogeneous integral equation. Various techniques have proposed in literature, based on rigorous (integral equations, transverse resonance methods) or approximate (Bloch wave analysis, transmission line analysis) methods. As done in [6], the algorithm adopted here is based on the solution of a resonance equation, obtained by imposing the matrix determinant of the infinite periodic array problem $det[Z]_\infty$ to be zero. The eigenvalues (i.e., the propagation wavenumbers) $k_\rho = \beta - j\alpha$ are obtained from the roots of the equation, with the constraint imposed by the generalized Fermat principle condition (4).

After the propagation wavenumbers are determined, the FIE is solved with a MoM scheme. Testing the field continuity with proper weight functions leads to a small-size linear system, whose solution provides the coefficients a_h and b_i of the expansion (5). It is worth noting that

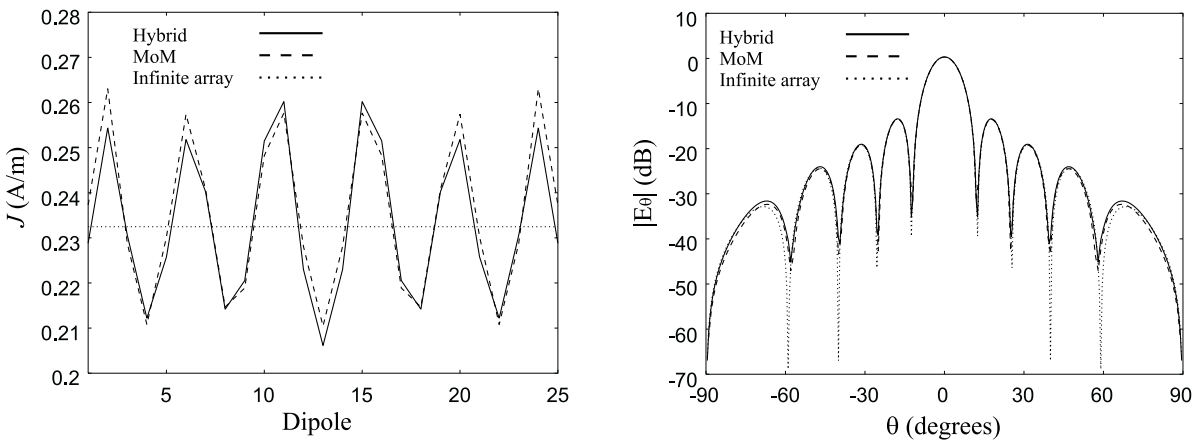

Fig. 2. (Left) Amplitude of the current on the central row of dipoles. (Right) Far-zone scattered electric field. Continuous line: hybrid asymptotic-MoM; dashed line: element-by-element MoM; dotted line: infinite periodic array approximation.

the number of basis functions in (5) is completely independent of the array size, since they are associated to the global diffraction mechanism at the array border.

To prove the effectiveness and the accuracy of the method, a numerical example is provided in Fig. 2. Results refer to a 25×25 array of printed dipoles ($L = 0.6$ mm, $W = 0.1$ mm) with periodicity $d_x = d_y = 0.8$ mm. The substrate has relative dielectric constant $\varepsilon_r = 10.2$ and thickness $d = 0.1905$ mm. The array is excited by a plane wave coming from broadside direction with electric field polarized along the dipoles. The operating frequency is $f = 7$ GHz. Fig. 2 shows the amplitude of the current on the central row of dipoles and the far-zone scattered electric field. Additional numerical results will be shown during the oral presentation.

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